Stat 414 - Day 17
Level 1 Predictor (Section 4.4)

Last Time:
- When have dependent observations, standard errors are likely too small
  - Could adjust standard errors using effective sample size
  - Multilevel models perform a similar adjustment
- Model equations (e.g., Level equations vs. composite equation) for random effects model
- In the multilevel framework, to assess significance of the grouping variable can
  - Look at traditional anova
  - Do a Likelihood ratio test
  - Look at confidence interval for variance parameter

Example 1: Netherlands language scores (see text)

The Netherlands Language dataset examines language test scores (langPOST) in Grade 8 students (~ age 11) for elementary schools in the Netherlands. (See p. 50 for more information about this dataset.) Students (Level 1) are nested within a random sample of schools (Level 2) and we will treat the schools as random effects.

Create the null model (using lmer)
```
#install.packages(lme4)
library(lme4)
nullmodel = lmer(langPOST ~ 1 + (1|schoolnr), data = neth, REML = FALSE)
#using ML to better match the output in the text
```
(a) Based on the above output, how many students are in the data set? How many schools are in the dataset?

$n = 3758$ # of schools = 211

(b) Using the null model, what do you predict for the language score of a randomly selected student? Is this the same as the mean language score in the dataset? Why or why not?

$\hat{\mu}$ vs $\bar{y}$ w/ unequal group sizes

(c) What is the estimated standard deviation of the language scores? Is this the same as the standard deviation of all the language scores in the sample? Why or why not?

$\sqrt{18.13 + 62.85} \approx 9$ vs. SD(y)   ICC = .224

As noted in HW 4, you can view the estimated random effects and calculate their standard errors. A "Catepillar plot" is a nice visual for sorting and visualizing the estimated effects.

(d) Is it reasonable to pick out the schools with the largest positive effects and conclude they are doing something better than the other schools? (*Hint*: This is more of an opinion question, check out http://www.amstat.org/asa/files/pdfs/POL-ASAVAM-Statement.pdf to learn about some of the controversy surrounding Value Added Models)

Now we want to include pupil (verbal) IQ as a predictor of language test performance.

(e) Is this a Level 1 or Level 2 predictor?

Level 1

(f) Write out an appropriate statistical "random intercepts" model (both composite and level equations) including verbal IQ. How many parameters are to be estimated?

$$y_{ij} = \beta_0 + \beta_1 \text{verbalIQ}_{ij} + u_j + \varepsilon_{ij}$$

Level 1: $y_{ij} = \beta_{0j} + \beta_1 \text{verbalIQ}_{ij} + \varepsilon_{ij}$ $\quad \varepsilon_{ij} \sim N(0, \sigma^2)$

Level 2: $\beta_{0j} = \beta_{00} + u_j$ $\quad u_j \sim N(0, \tau^2)$

Include the IQ variable (which has been centered (though before students with missing values were removed)) in the model.

```
model1 = lmer(langPOST ~ 1 + IQ_verb + (1|schoolnr), data = neth, REML=F)
summary(model1)
confint(model1)
```

(g) How many/Which parameters are estimated by this model? 4

(h) Provide interpretations of the estimated slope and intercept.

$\hat{\beta}_1 = 2.507$ predicted ↑ in lang score w/ 1 unit ↑ in IQ in a particular school

$\hat{\beta}_{00} = 41.05$ predicted lang w/ IQ at mean at avg school

(i) What is the estimated variation in responses for a particular value of IQ_verb?

tau^2 + sigma-hat^2 is now 9.845 + 40.469 = 50.314

(j) Which has changed more, the estimated within-group variation or the estimated between-group variation? Does this make sense in context? Is it possible for both of them to decrease? What does that mean?

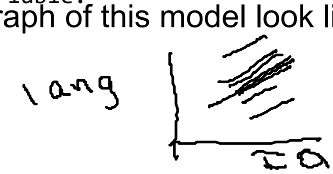
Could say, we changed tau^2 from 18.13 to 9.85 (about half) and sigma^2 from 62.85 to 40.469, so the between group variation changed a bit more. A little surprising how much the Level 2 variation decreased from adding a Level 1 variable. Must be some IQ differences across the schools.

(k) What is the new value of the ICC? How do you interpret this? What would it mean for this value to be super close to zero?

```
performance::icc(model1)
```

Adjusted ICC = 9.845/(9.845 + 40.469) = 0.196 It's a little smaller as there is less group-to-group variation now. If zero, then would say we had explained all of the group to group variation by adding this variable.

(l) What would a graph of this model look like? What if we had treated the schools as fixed effects?



The lines for fixed effects would be more spread out with no shrinkage to the overall line

(m) Is the (conditional) effect of verbal IQ statistically significant? How are you deciding?

Yes, the t-value is 7.928 > 2

(n) What if we had ignored the grouping by class?

```
summary(lmmodel <- lm(neth$langPOST~neth$IQ_verb))
```

The new slope of verbal IQ is 3SE from first slope!
This intercept SE is twice as large. So there are some meaningful changes to the model when we ignore an important variable/clustering.

A neat graph showing the fitted lines:

```
preds = predict(model1, newdata = neth)

ggplot(neth, aes(x = IQ_verb , y = preds , group = schoolnr, color = schoolnr )) +
geom_smooth(method = "lm", alpha = .5, se = FALSE) +
geom_jitter(data = neth, aes(y = langPOST), alpha = .1) +
  theme_bw()

#fixed effects
model1F = lm(langPOST ~ 1 + IQ_verb + factor(schoolnr), data = neth)

predsF = predict(model1F, newdata = neth)

ggplot(neth, aes(x = IQ_verb , y = predsF , group = schoolnr, color = schoolnr )) +
geom_smooth(method = "lm", alpha = .5, se = FALSE) +
geom_jitter(data = neth, aes(y = langPOST), alpha = .1) +
  theme_bw()
```

Notes

- It's probably a good idea to grand mean center all explanatory variables before you start your analysis.
- The adjusted intraclass correlation coefficient is often smaller than the "raw" (null model) intraclass correlation coefficient.
- Performance package: The adjusted ICC is what we would calculate "by hand" which just uses the variance components after adding the covariate into the model. The unadjusted ICC takes "fixed effect" variance into account (in the denominator) as well (see insight::get_variance(model)) (the change in unexplained variation when the fixed effect is added to the model). We will focus more on the adjusted ICC, if that. Of real interest to us is the unadjusted ICC from the null model, but you can look at the ICC in other models to see how that has impacted the "unexplained" group to group variation.

  Reference: Nakagawa S, Johnson P, Schielzeth H (2017) The coefficient of determination R2 and intra-class correlation coefficient from generalized linear mixed-effects models revisted and expanded. J. R. Soc. Interface 14. doi: 10.1098/rsif.2017.0213

To think about: - Are these nested models? What would the likelihood ratio test tell us?