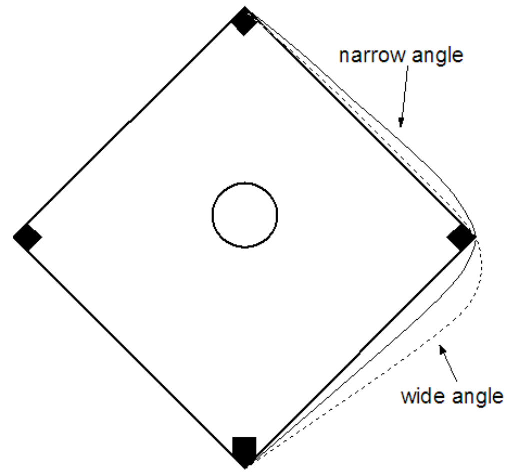
Stat 301 – Day 34
Paired Designs (Inv 4.8)

Does the path that you take to "round" first base make much of a difference? Hollander and Wolfe (1999) report on a Master's Thesis by W. F. Woodward (1970) that investigated different base running strategies. For example, you could take a "narrow angle" or a "wide angle" around first base. In Woodward's study, he used a stopwatch to time runners going from a spot 35 feet past home to a spot 15 feet before second based. Each of 22 runners ran both routes, in random order.

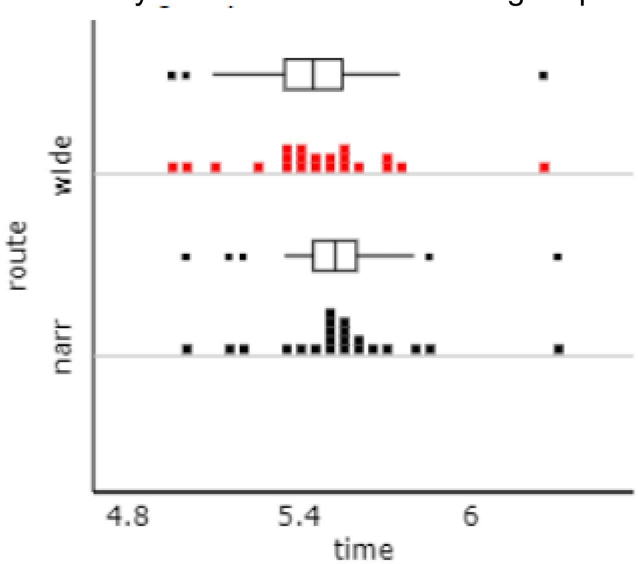


(a) Identify the observational/experimental units.

(b) Identify the explanatory and response variables. Classify each as quantitative or categorical.

(c) Is this a *paired design* or an *independent samples design*? Explain.

(d) What do you learn from the following output?



Summary statistics:

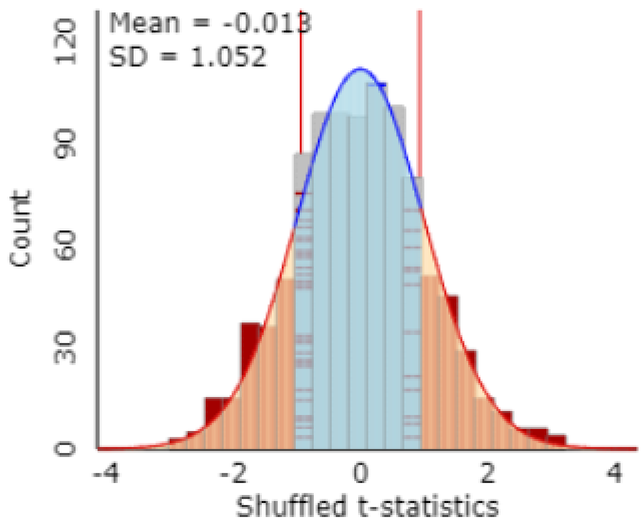
	n	Mean	SD
wide	22	5.46	0.27
narr	22	5.53	0.26
pooled	44	5.50	0.27

Observed Mean Group Diff = 0.075

(e) What do you learn from the following output?

Statistic: t-statistic

Total Shuffles = 1000



Count Samples Beyond .93

Count = 367/1000 (0.3670)

☑ Overlay *t* distribution

theory-based p-value=0.3577, df = 41.90

(f) Explain why the analysis in (e) is completely incorrect!

(g) Open the RCBD applet and use the pull-down menu to select the Base Running dataset.

- Press the Use Data button.
- Check the Show Shuffle Options box
- Enter 1000 for the number of shuffles and press the Shuffle Responses box

Confirm that the distribution of the distribution of sample means is approximately normal with mean close to zero and standard deviation close to $\sqrt{\frac{.273^2}{22}+\frac{.260^2}{22}}$ = 0.080 (and verify the t-statistic used above).

This simulation approach does not match the randomness in the study design. Another way to model the null hypothesis being true, and matching the randomness in the study design, is to keep the observations the same, but simulate the randomization of which observation is "wide angle" and which observation is "narrow angle."

(h) Change the number of shuffles to 1 and select the "within blocks" option. Press Shuffle Responses. Describe what the simulation is doing instead. Conjecture how this is going to impact the distribution of the difference in means.

(i) Change the number of shuffles to 1000 and press Shuffle Responses. Was your conjecture correct? How does this impact the p-value? The confidence interval?

Recall that when we have correlated observations, we need to consider the correlation in the observations when estimating the standard error of the statistic. Consider

$$SD(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} - 2\rho\frac{\sigma_1\sigma_2}{\sqrt{n_1 n_2}}}$$

Software tells us that the correlation coefficient between the two sets of observations is 0.946.

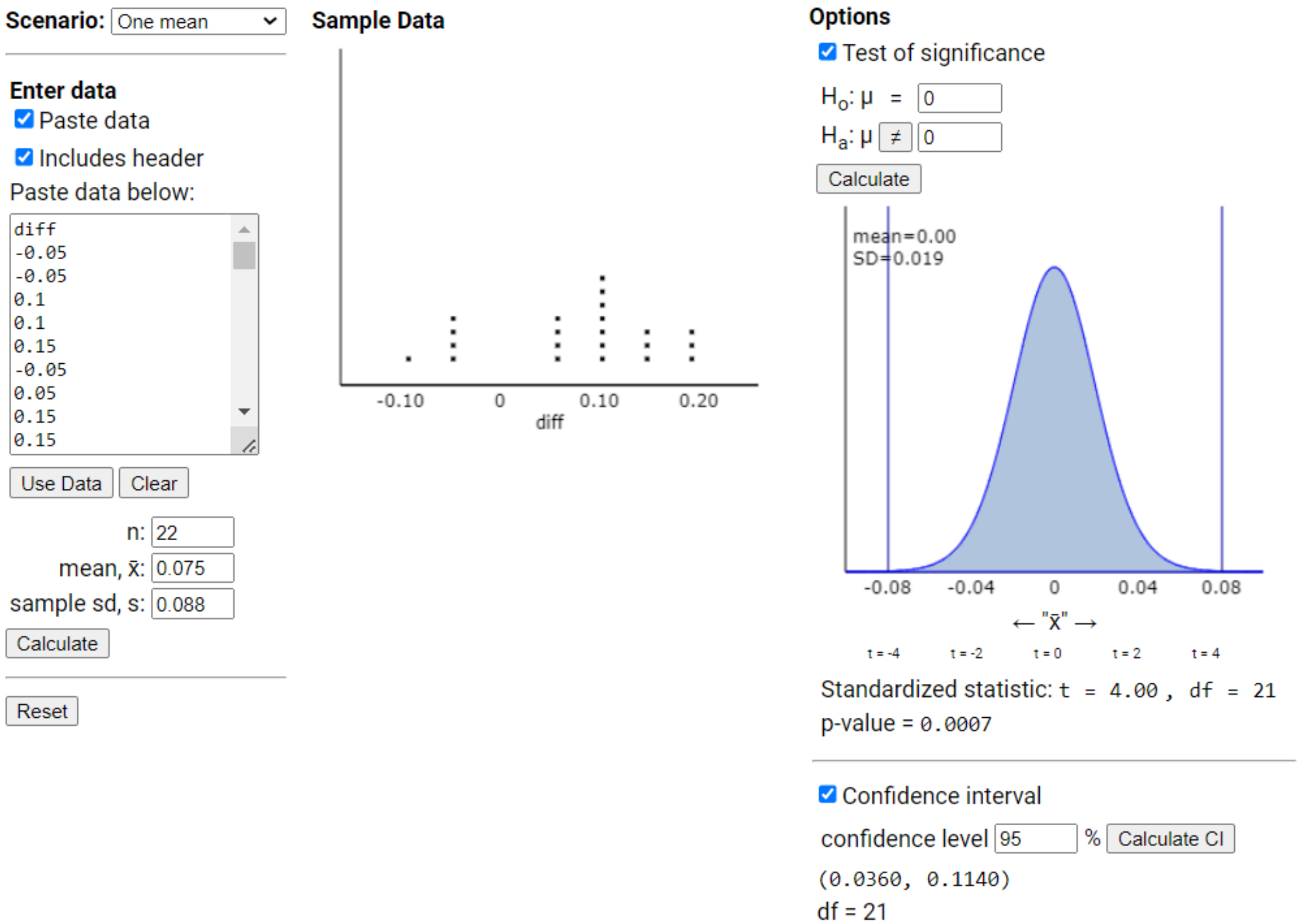
(j) Does this positive correlation increase or decrease the standard error of the statistic.

In fact, for a paired design ($n_1 = n_2$), we can estimate this with $SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{n}}$ which turns out to equal $\frac{s_{diff}}{\sqrt{n}}$ where s_{diff} is the standard deviation of the differences. So our test statistic will be

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \delta_0}{\sqrt{\frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{n}}} = \frac{\bar{x}_{diff} - \delta_0}{\frac{s_{diff}}{\sqrt{n}}}$$

Key Idea: To analyze paired data, we will first compute the differences in each pair, and then use a one-sample *t*-test of the differences.

Consider the following output for the differences (*narrow* – *wide*)



(k) Compute $\frac{s_{diff}}{\sqrt{n}}$ and compare to the simulation results. Why is it not a perfect match?

(l) Compute the *t*-statistic. What do you conclude?

(m) Interpret the confidence interval in context.

(n) In this study, the participants are called "blocks." Check the Adjust data for block effects box, does the difference in treatments appear statistically significant now?