Stat 414 – Day 8
Random Effects and Model Diagnostics

between group var

Last Time: Estimating Random Effects

- Random intercepts model $Y_{ij} = \underbrace{\beta_0}_{\text{fixed}} + \underbrace{u_j}_{\text{random}} + \epsilon_{ij}$ $u_j \sim N(0, \tau^2)$, $\epsilon_{ij} \sim N(0, \sigma^2)$ within group var
  - Intercept is the expected (population mean) response for average subject
  - $\hat{\beta}_0 = \bar{y}$ if balanced or avg of $\bar{y}_j$
  - Fixed effect: $\hat{u}_j = \bar{y}_j$ (but then use pooled standard error)
  - Random effect: $\hat{u}_j = w_j\bar{y}_j + (1 - w_j)\bar{y}$ ("shrinkage estimator") where $w_j = \tau^2 / (\tau^2 + \sigma^2/n_j)$ (aka the "reliability" of group j)

So then what is the standard error of $\hat{u}_j$? Can we find a confidence interval around this estimator? Do we want to? Questionable

Fixed effect:

Confidence interval for Jones: μ_{Jones}

model1$fitted.values[bball$Player == "Jones"][1]	= .37957 – 0.17730 = 0.2023
new.dat <- data.frame(playerf="Jones") predict(model1, newdata = new.dat, interval = 'confidence')	.2023 ± t*(37)(.0448/sqrt(11)) = (.1749, .2296) $\mu + \alpha_{jones}$

Confidence interval for Jones' "effect":

Coefficients:
`            Estimate Std. Error t value Pr(>|t|)`
`(Intercept)  0.379571   0.007523  50.453  < 2e-16 ***`
`playerf1    -0.084571   0.016722  -5.058 1.18e-05 ***`
`playerf2    -0.177298   0.013351 -13.280 1.19e-15 ***`
`playerf3    -0.056237   0.016722  -3.363   0.0018 **`
`playerf4     0.170429   0.016722  10.192 2.72e-12 ***`
`playerf5    -0.022753   0.013351  -1.704   0.0967 .`
	-.177298 ± 2.026(.01335) = (-.204, -.150)
	α_{jones}

Random effect:

```
Random effects:
 Groups   Name        Variance Std.Dev.
 Player   (Intercept) 0.019648 0.14017
 Residual             0.002008 0.04481
Number of obs: 43, groups:  Player, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)  0.37885    0.05772   6.564
```

ranef(model2)

```
$Player
          (Intercept)
Anderson  -0.08244295
Jones     -0.17494910
Mitchell  -0.05458408
Rodriguez  0.16828693
Smith     -0.02182608
Suarez     0.16551528
```

Confidence interval for Jones' "effect":

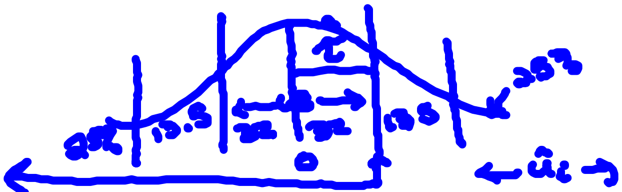
Comparative standard error for u_j = $\sqrt{\dfrac{1}{\frac{1}{\tau^2} + \frac{n_j}{\sigma^2}}}$

$\sqrt{\dfrac{1}{\frac{1}{.0195} + \frac{11}{.002008}}} = .0135$

```
ranef(model2, condVar= TRUE, drop=TRUE)
$Player
   Anderson        Jones
-0.08244295 -0.17494910
attr(,"postVar")
[1] 0.0003290190 0.0001808414
```

$.01345^2$

-.174949 ± 2.026(.01345) = (-.2022, -.1477)



Partly, you are more interested in the distribution of effects, rather than these individual players. We have estimated that this distribution is approximately normal with mean 0 and variance 0.01965.

(a) What is the expected average for a player in the 84th percentile?

mean + 1 SD
.1407 => .3796 + .1402 = .5198

So would we even look at these random effects estimates?

(b) What assumptions have we made about these "level 2 effects"?

normally distributed

We will add checking these residuals as part of our model diagnostics (Section 10.6).

- Check standardized level 2 residuals for normality though doesn't always guarantee real effects follow a normal distribution. Also check for unusual observations.
- Plot against the Level 2 units and other Level 2 variables (e.g., nonlinearity)
- Plot squared residuals against Level 2 variables to check for heteroscedasticity

(c) Try > plot(model2) and >plot(ranef(model2))

Example 2: (Example 4.1 in text): We want to predict language test scores (langPOST) in Grade 8 students (~ age 11) in elementary schools in the Netherlands based on their (verbal) IQ.

(a) Identify the fixed and random effects in this context. Identify level 1 and level 2. (units)

Level 1 : students
Level 2 schools

lang score - fixed
IQ - fixed
schools - random

(b) Use R to create the null model. How many students and how many schools are in the dataset? Does this model appear to be valid?

3758 211

(c) Using the null model, what do you predict for the language score of a randomly selected student?

41.0038 vs. 41.4 = $\bar{y}$

(d) What standard error would you put around your estimate? Is this the same as the standard deviation of all the language scores in the sample? Why or why not?

$\pm\sqrt{18.24 + 62.85} \approx 9.036$ vs. 8.89 SD(Y)

(e) Is it reasonable to pick out the schools with the largest positive residuals and conclude they are doing something better than the other schools?

larger u_j

confounding variables?

(f) Now we want to include pupil (verbal) IQ as a predictor of language test performance. Is this a Level 1 or Level 2 predictor?

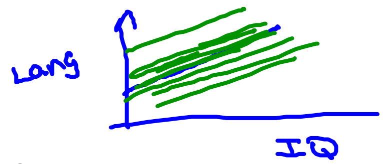
Level 1

(g) Write out an appropriate statistical "random intercepts" model. How many parameters?

$$Y_{ij} = \underline{\beta_0} + \beta_1 IQ_{ij} + \underline{u_j} + \varepsilon_{ij}$$

(h) Include the IQ variable in the model and provide an interpretation of the slope and intercept from the parameter estimates. What would a graph of this model look like? Does this model appear to be valid?

$$\hat{Y}_{ij} = 41.06 + 2.51\, IQ \quad \text{"fixed"}$$



(i) Is the effect of IQ statistically significant?

$t = 46.11 > 2$ so small p-value

(j) What is a "typical deviation" in these regression lines from the overall regression line? Are these distances negligible? What is a "low line"? What is a "high line"?

typical deviation = standard deviation

$= 3.138 = \hat{\tau}$

(k) What is a "typical deviation" of a student from his/her school regression line? More or less than between the lines?

$6.362 = \hat{\sigma}$

more than between lines

(l) What is the "residual" intraclass correlation coefficient?

$$\frac{3.138^2}{3.138^2 + 6.362^2} = .196$$

(l) How do the residual variance and the random intercept variance compare to the empty model? How do the likelihood and AIC/BIC values compare? Telling you?

null model

$\hat{\tau}$ 4.27 ↓ 3.138 also explained some school to school var

$\hat{\sigma}$ 7.93 ↓ 6.36 we added a student level variable that explained student to student variation

(m) Now that we have "controlled for IQ," can we pick out the schools with the most positive random effects and declare them superior?

probably not the best idea

Model for hurricane data (HW 3) effect coding for hurricane gender:

$$Y_{ij} = \beta_0 + \beta_1 \times \text{name.gender}_{ij} + u_j + \epsilon_{ij}$$

- Intercept: Expected rating for "average hurricane" for average subject
- $2\beta_1$ = average rating for females – average rating for males, adjusted for subject
- With lmer, confint gives you confidence intervals for the fixed and random components (with lme, use intervals(), results will vary slightly)

Approach 1: Paired analysis on the means (diff = avg.male – avg.female)

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.2257	0.0497	4.541	0.00000773	***

We are 95% confident that the mean rating is 0.128 to 0.323 higher for male names compared to female names.

Approach 2: Mixed model with hurricane-name-gender as fixed effect (1 = female, 0 = male)

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	4.38555	0.04472	98.073
HurrGend	-0.22572	0.04255	-5.305

HurrGend	-0.3091269	-0.1423182

Approach 3: Ignore subject/treat all observations as independent

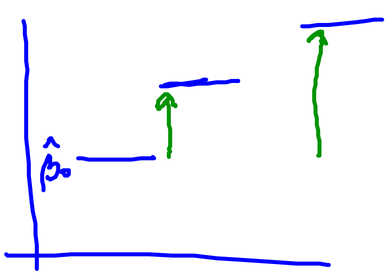
Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	4.385549	0.03351924	130.83675	0
HurrGend	-0.225723	0.04740337	-4.76174	0

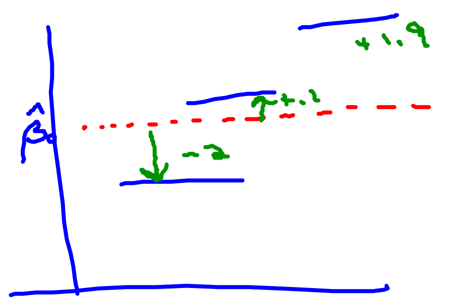
```
> confint(gls(Score ~ HurrGend , data = hurr_long))
                 2.5 %     97.5 %
(Intercept)  4.3198526  4.4512456
HurrGend    -0.3186314 -0.1328136
```

All these approaches are assuming the hurricane-name-gender effect is the same across the subjects, but that there is subject-to-subject variation in ratings.

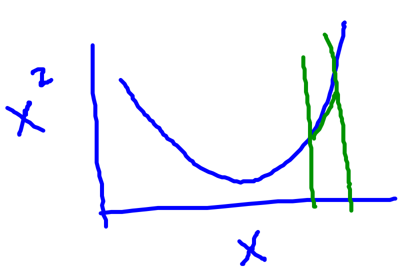
Also see new review problem.

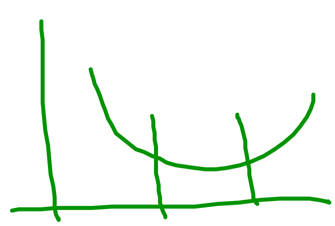


indicator



effect





after centering