ISCAM 2: CHAPTER 1 EXERCISES

1. Go Take a Hike!
The book *Day Hikes in San Luis Obispo County* by Robert Stone gives information on 72 different hikes that one can take in the county. For each of the 72 hikes, Stone reports the distance of the hike (in miles), the anticipated hiking time (in minutes), the elevation gain (in feet), and the region of the county in which the hike can be found (North County, South County, Morro Bay, and so on, for a total of eight regions).
(a) What are the observational units here?
(b) How many variables are there?
(c) Classify each variable as quantitative or categorical.
(d) If you create a new variable that is the ratio of hiking time to hiking distance, would that be a quantitative or categorical variable?
(e) If you create a new variable called *length of hike* that is coded as "short" for a hike whose distance is less than 2 miles, "medium" for a hike whose distance is at least 2 but not more than 4 miles, and "long" for a hike whose distance is at least 4 miles, would this be a quantitative or categorical variable?
(f) For each of the following, specify whether or not it is a legitimate variable for the observational units you specified in (a):
   •   Longest hike in the book
   •   Average elevation gain of the hikes in the book
   •   Whether or not the hike is in the Morro Bay region
   •   Proportion of hikes with an elevation gain of more than 500 feet
   •   Anticipated hiking time, reported in hours

2. Identifying Variables
(a) Consider the 50 states as observation units for a study. Suggest one quantitative and one categorical variable that you could measure about the states. (Be careful to express you answers clearly as variables.)
(b) Consider the nine members of the Supreme Court as the observational units for a study. Suggest one quantitative and one categorical variable that you could measure about the justices. (Be careful to express your answers clearly as variables.)

3. Bookstores
Consider all of the books in a bookstore as the observational units in a study.
(a) Identify three quantitative variables that could be recorded on these books.
(b) Identify three categorical variables that could be recorded on these books.

4. Rock n' Roll
We saw a claim on "trivia coaster" from the Rock n' Roll Hall of Fame that 30% of all songs with a color in the title are about the color blue.
(a) Clearly state in words the observational units and the variable that this claim refers to.
(b) Is the variable quantitative or categorical?
(c) Clearly state in words the population and parameter that this claim refers to.
(d) Clearly state, both in words and in symbols, the null and alternative hypotheses for testing the claim.

5. Predicting Elections from Faces?
Do voters make judgments about political candidates based on his/her facial appearance? Can you correctly predict the outcome of an election, more often than not, simply by choosing the candidate whose face is judged to be more competent-looking? Researchers investigated this question in a study published in *Science* (Todorov, Mandisodka, Goren, & Hall, 2005). Participants were shown pictures of two candidates and asked who has the more competent looking face. Researchers then predicted the winner to be the candidate whose face was judged to look more competent by most of the participants. For the 32 U.S. Senate races in 2004, this method predicted the winner correctly in 23 of them.
(a) In what proportion of the races did the "competent face" method predict the winner correctly?
(b) Describe (in words) the null model to be investigated with this study.
(c) Describe how you could (in principle) use a coin to produce a simulation analysis of whether these data provide strong evidence that the "competent face" method would correctly predict the election winner more than half the time. Include enough detail that someone else could implement the full analysis and draw a reasonable conclusion.
(d) Use the One Proportion Inference applet to conduct a simulation (using 1000 repetitions), addressing the question of whether the researchers' results provide strong evidence in support of the researchers' conjecture that the "competent face" method would correctly predict the election winner more than half the time. Submit a copy of the applet output (e.g., "print screen" key or a screen capture), and indicate where the observed research result falls in that distribution. Also report the approximate p-value from this simulation analysis.
(e) Write a paragraph, as if to the researchers, describing what your simulation analysis reveals about whether the data provide strong evidence in support of their conjecture.
These researchers also predicted the outcomes of 279 races for the U.S. House of Representatives in 2004. The "competent face" method correctly predicted the winner in 189 of those races.
(f) Use the applet to conduct a simulation analysis of these data. Again submit a copy of the "what if?" distribution, and indicate where the observed research result falls in that distribution. Also report the approximate p-value, and summarize your conclusion, again as if to the researchers.

6. Can Dolphins Communicate?
A famous study from the 1960s explored whether two dolphins (Doris and Buzz) could communicate abstract ideas. The dolphins were trained to push the right button if a headlight was shone steadily, but the left button if the headlight blinked on and off. Then the researcher placed a large wooden wall in the middle of the pool. Doris was on one side of the wall and could see the headlight, whereas Buzz was on the other side of the wall where he could not see the headlight. When the light was shone, Doris would swim near the wall and whistle. Buzz would then whistle back and press a button. If he pushed the correct button (corresponding to the light Doris was shown), they both got a fish. Dr. Bastian repeated this procedure again and again, and Buzz pushed the correct button 22 out of 25 times. Is this convincing evidence that Buzz and Doris could communicate?
(a) Calculate the statistic for this study.
(b) State the null and alternative hypotheses (in symbols and in words) to investigate this research question.
(c) Use technology to calculate an exact p-value for this significance test. Include a copy of your computer results (e.g., screen capture).
(d) Write a one-sentence interpretation of this p-value.
(e) Summarize the conclusions you would draw about the research question based on this p-value.

7. Which Tire?

A statistics class at Cal Poly collected data on a well-known campus legend. Each student was asked to specify one of the four tires to answer in a situation where you have to make up which tire had recently been flat on your car. The prior conjecture is that a higher number than would be expected due to chance alone would pick the *right front* tire. In this class, 24 of 54 students in class chose the right front tire (a tire identified in advance as being one that people tend to pick out of the four). You will conduct a test of whether these data provide evidence that Cal Poly students tend to choose the right front tire more often than would be expected if the four tire choices were equally likely.

(a) Identify the observational units and variable in this study. Also classify the variable as categorical or quantitative. If the variable is categorical, also indicate whether it is binary.
(b) State the appropriate null and alternative hypothesis, in symbols and in words.
(c) Use technology produce a bar graph of the student responses. Submit this graph, and comment on what it reveals.
(d) Use technology to determine the (exact binomial) p-value for the test of your hypotheses in (b).
(e) Write a sentence describing what this p-value is the probability of.
(f) Write a couple of sentences summarizing the conclusion that you would draw from this analysis and also explaining the reasoning process that underlies your conclusion.
(g) Suppose that another statistics class conducts this same study in their own class, which has exactly half as many students. Suppose further that this class obtains the same proportion of students choosing the right front tire. Determine the exact p-value in this case. Describe how the p-value and your conclusion would differ for this class of 27 students compared to the first class of 54 students, and comment on why this makes intuitive sense.

8. Which Tire? (cont.)

Reconsider Exercise 7, where you tested whether sample data suggest that more than 25% of a population would answer "right front" when asked to name a tire that had gone flat. Suppose that you read of a study in which 30% of a random sample answered "right front."

(a) What further information would you require to assess whether this sample result constitutes strong evidence that more than 25% of the population would answer "right front"? Also explain why this information is needed.
(b) Determine the p-value of the test for the following sample sizes, in each case supposing that the sample proportion answering "right front" is 0.3: $n = 10, 50, 250, 500$. (Feel free to use technology, but explain what you ask the technology to do in each case.)
(c) Determine the smallest sample size for which this sample result would be statistically significant at the 0.05 level.
(d) Repeat (c) for the 0.01 level.

9. Rock-Paper-Scissors

For informal sports events, players often play "rock-paper-scissors" to decide who serves first or who is the home team etc. Two players simultaneously show one of the three objects. The player showing the dominant object (e.g., rock beats scissors) wins. The optimal strategy is to alternate among the three objects. Play the game rock/paper/scissors against the computer using the website: http://www.nytimes.com/interactive/science/rock-paper-scissors.html. Select the novice version of the computer to play against. Play for at least 30 rounds, but keep going for as long as you'd like. Keep track of which option you choose (rock or paper or scissors) for every round that you play (the computer will record this for you but that information will soon scroll off the screen, so make your own notes). *Try to recreate how you would play against a person and don't view your prior results when making your next selection.*

(a) Identify the observational units in this study.
(b) Identify the variable of interest.

An article published in *College Mathematics Journal* (Eyler, Shalla, Doumaux, & McDevitt, 2009) found that players tend to not prefer scissors, choosing it less than 1/3 of the time. We will investigate whether your data suggest that *you* tend to choose scissors less than one-third of the time.

(c) Calculate the statistic in this study and create a bar graph of your results. (In R and Minitab, you can do this with just the "summarized" data, you don't have to enter the individual outcomes.) Are your results in the direction conjectured by these researchers (choosing scissors less than 1/3 of the time)?
(d) Define the parameter of interest in this study.
(e) State appropriate null and alternative hypotheses about this parameter according to the theory suggested in the *CMJ* article.
(f) Explain how you could use an ordinary six-sided die to simulate a what-if distribution under this null hypothesis. Be sure to indicate what each possible outcome of the die (1, 2, 3, 4, 5, 6) would represent.
(g) Based on your sample results, are you convinced that you choose scissors less than one-third of the time in the long run? Clearly explain your reasoning.

10. Plausible Values?
In Investigation 1.3, you considered the sample of 361 patients with 71 deaths and whether 0.15 was a plausible value for the probability of a death during a heart transplantation at St. George's hospital at this time. Use the One Proportion Inference applet to test other values for π (e.g., 0.20, 0.21, 0.22, …). Create a list of which values (multiples of .01 are fine) that have a *two-sided* p-value of at least 0.05. That is, what are the *plausible* values of π based on the observed sample result? Include a screen capture of your applet results for the smallest value of π you consider plausible and the largest value of π you consider plausible.

11. Competitive Advantage of Uniform Color?
Does uniform color give athletes an advantage over their competitors? To investigate this question, Hill and Barton (*Nature*, 2005) examined the records in the 2004 Olympic Games for four combat sports: boxing, tae kwon do, Greco-Roman wrestling, and freestyle wrestling. Competitors in these sports were randomly assigned to wear either a red or a blue uniform. The researchers investigated whether competitors wearing one color won significantly more often than those wearing the other color. They analyzed results for a total of 457 matches. Of these, red won the match 248 times, while blue won 209 times.

(a) Identify the observational units and binary, categorical variable of interest. Indicate which outcome you will consider "success."
(b) Explain how and why randomness was used in this study.
(c) State the null and alternative hypotheses based on this research question. [*Hint*: Before you look at the data.]
(d) Calculate a binomial p-value for investigating this research conjecture.
(e) Compute a 95% binomial confidence interval for the parameter. Write a sentence interpreting what this interval says, including how you are defining the parameter.
(f) Is the confidence interval result consistent with your p-value result? Explain.
(g) Now determine a 90% confidence interval for the parameter. Comment on how it differs from the 95% interval. [*Hint*: Refer to both the midpoints of the intervals and their widths.]
(h) Summarize your results as if to an athletic director at a university. Include discussion about how you are willing to "generalize" these results beyond these 457 matches.

12. Competitive Advantage of Uniform Color (cont.)

For the Competitive Advantage of Uniform Color data, assuming $\pi = 0.5$:
(a) State the appropriate null and alternative hypotheses, both in symbols and in words.
(b) Use the One Proportion Inference applet to simulate 1000 repetitions of these matches, under the null hypothesis. Submit a screen capture of the applet results.
The researchers found that the competitor wearing red defeated the competitor wearing blue in 248 matches, and the competitor wearing blue emerged as the winner in 209 matches.
(c) Use the applet simulation results to approximate the two-sided p-value from these data. Also report which values are being counted to determine this approximate p-value.
(d) Use technology to determine the exact binomial (two-sided) p-value. Submit the output with your answer.
(e) Summarize what your analysis reveals about how much evidence the data provide for concluding that uniform color does give one athlete an advantage over the other.
(f) Use technology to determine a 95% confidence interval for the parameter. Also write a sentence interpreting what this interval says.
(g) Now determine a 99% confidence interval for the parameter. Comment on how it differs from the 95% interval. [*Hint*: Refer to both the midpoints of the intervals and their widths.]
(h) Are these confidence intervals consistent with your earlier test (parts a-e)? Explain briefly.

13. A Cup of Tea

One of the first descriptions of a true randomized experiment was given by Ronald Fisher in 1935 (*The Design of Experiments*). He described a tea party in Cambridge where a woman claimed that she could tell whether a cup of tea with milk had the milk added to the tea or the milk poured into the cup first and then the tea added. Fisher proposed an experiment to determine whether she truly could tell the difference.
(a) Explain how randomization could be used in such an experiment.
(b) If she is given one cup of tea, what is the probability that she could give the right answer even if she is only guessing?
(c) Suppose she will be given 10 cups. Let X represent the number of cups for which she makes the correct identification of which ingredient was poured first. Can X be considered a binomial random variable? Explain.
(d) What are the parameters of the binomial distribution in (c)? Explain why the exact value for one of the parameters is unknown.
(e) Suppose that she is just guessing on each cup and also that she doesn't know how many cups there are of each type. Use the One Proportion Inference applet to determine how many identifications she needs to get correct out of ten so that the probability of doing so by guessing is less than 0.05.
(By the way – Fisher never said how many she did get correct!)

14. Binomial or Not?

A binomial random variable counts the number of successes with a fixed probability of success and a fixed number of independent trials. Explain why the following are *not* binomial random variables.
(a) Let X = the number of computer solitaire games you play before your first win.
(b) Let Y = the time you wait in line at a convenience store each time you visit.
(c) Let W = the number of long words in a sample of ten words from the Gettysburg Address.

15. Binomial Properties
Consider a random variable X having a binomial distribution with parameters n and π.
(a) For a fixed value of n, for what value of π is E(X) maximized? [*Hint*: Remember the restriction on possible values of π.]
(b) For a fixed value of n, for what value of π is SD(X) maximized? [*Hint*: Use calculus.]
(c) For a fixed value of n, for what two values of π is SD(X) minimized? [*Hint*: You should not need to use calculus.]

16. Pilot Study
On a recent trip one of the authors took four flights and encountered eight pilots (two per flight). Seven of these eight pilots were men, and one was a woman. Let the random variable X represent the number of women in a random sample of 8 commercial pilots.
(a) Would it be reasonable to model the distribution of X with a binomial distribution? Explain and clearly define the parameters of the distribution in words.
(b) Suppose that half of all commercial pilots are women. Determine the probability of encountering one or fewer women in a random sample of eight commercial pilots. (Show the details of your calculations (technology output) to support your answer.)
(c) Is the probability in (b) small enough to convince you that, based on these sample data, fewer than half of all commercial pilots are women, using the 0.05 level of significance?
(d) According to Women in Aviation International (WIA), women comprised 5% of all commercial pilots in 2001. Use this parameter value to calculate and graph the probability distribution of X. [*Hint*: List all possible values and their probabilities. You may use technology.]
(e) Assuming that the WIA parameter value is correct and using the results from (d), is *one* the most common value for the number of women in a random sample of eight commercial pilots? Would you say that it is a surprising value? Explain.

17. Pilot Study (cont.)
Reconsider the previous question, about encountering one woman in a sample of eight commercial pilots. Let X be a binomial random variable with parameters $n = 8$ and π = proportion of all commercial pilots that are female.
(a) Express $P(X = 1)$ as a function of π.
(b) Graph this function, for all possible values of π from 0 through 1.
(c) Use calculus to determine the value of π that maximizes this $P(X = 1)$ function. Verify from the graph that this value of π really does maximize the function.
(d) Explain why this value of π that maximizes this $P(X = 1)$ function makes intuitive sense.
(e) More generally, use calculus to determine the value of π that maximizes the function $P(X = k)$, where k can be integer from 0 through 8, inclusive. Does your answer make intuitive sense? Explain. (This question introduces you to a widely used statistical technique for estimating unknown parameters, known as maximum likelihood estimation.)

18. Rolling Die
Suppose that your instructor rolls a pair of dice five times.
(a) What is the probability that the sum of the dice would be either 7 or 11 all five times, assuming that the dice were fair? [*Hint*: Let the random variable X represent the number of rolls in which the sum is 7 or 11, and use the binomial distribution to calculate this probability. You will first need to

determine π, the probability that the sum would be 7 or 11 on any one roll by listing the sample space of 36 equally likely outcomes. Then use that probability for the binomial parameter.]
Show the details of your calculations, or technology output, to support your answer.
(b) Would this result (five consecutive rolls of 7 or 11) constitute reasonably strong evidence that the dice were not fair? Explain your reasoning, based on your answer to (a).
(c) Determine the probability of obtaining a sum of 7 or 11 in n consecutive rolls of a pair of fair dice as a function of n, where n is a positive integer.
(d) Graph this probability as a function of n, and comment on the behavior of the function.
(e) Determine the smallest value of n for which this probability dips below 0.05. Then find the smallest such values of n for which the probability dips below 0.01, 0.001, and 0.00001.

19. Winning Survivor

Contestants compete for a million dollar prize on the hit reality game show *Survivor*. For the first few years, when the competition had been weaned down to just three remaining contestants, one of them had won "immunity" and got to decide which of the other two will advance to the final round. During that final round, each of the two finalists presents his/her case to the "jury" on why he/she deserves to win the million dollars. In the first eight series of *Survivor*, the person of the final two who had immunity won the overall game only twice. For this question you will consider whether these results provide convincing evidence that the person with immunity is the ultimate winner (of the final two) less often than we would expect by chance.
(a) Define the parameter of interest in words.
(b) State the appropriate null and alternative hypotheses (in symbols and in words).
(c) Let X represent the number of contestants with immunity who go on to become the ultimate winner (of the final two). Can X be considered a binomial random variable? Explain.
(d) Calculate the p-value for the hypotheses stated in (b). Show the details of your calculations (or technology output) to support your answer.
(e) Is this p-value statistically significant at the 5% level?
(f) Do the data provide convincing evidence that the contestant with immunity becomes the ultimate winner less often than we would expect by chance? Explain, including any cautions that you have about generalizing these results to the general process.
(g) Suppose that none of those with immunity had gone on to win the overall game. How surprising would this result be if those with immunity did not have a disadvantage in the final two? Support your answer with an appropriate probability calculation.

20. Multiple-Choice

One of the authors moved to California in 2001 and faced the traumatic situation of taking a multiple choice exam to qualify for a California driver's license. This exam consisted of 36 multiple-choice questions, with three options provided for each question. Candidates needed to answer at least 31 questions correctly in order to pass. Let the random variable X be the number of questions answered correctly on the exam.
(a) Suppose that the candidate guesses randomly among the three options on each question. What probability distribution does X have? (Specify the parameters of the distribution as well as its name.)
(b) If the candidate guesses randomly on each question, what would be the expected number of questions that he would answer correctly? What would the standard deviation of the number correct be?
(c) If the candidate guesses randomly on each question, what is the probability of passing the exam? (Show the details of your calculations, or technology output, to support your answer, here and throughout.)

[*Minitab note*: You may need to double click on the X-axis to change the max value in order to see the results.]

(d) If the candidate can eliminate one wrong option on each question and guess randomly between the other two, what is the probability of passing the exam? (Identify the probability distribution of X in this case also.)

(e) If the candidate studies to the point of having a 0.9 probability of answering each question, independently from question to question, what is the probability of passing the exam? (Identify the probability distribution of X in this case also.)

(f) Let π represent the probability of answering each question correctly, independently from question to question. If the candidate wants to have at least a 99% chance of passing the exam, what is the smallest value of π that will achieve this? [*Hint*: Use trial-and-error, and report π to two decimal places.]

The rules actually allow a candidate to take the exam three times, and passing (with at least 31 of 36 correct) at least once is sufficient to qualify for a license.

(g) Suppose now that the candidate studies to the point of having a 0.8 probability of answering each question, independently from question to question. What is the probability of passing the exam at least once in the three allotted attempts?

21. Football Playoffs

An ESPN sports commentator reported that in the 15 cases prior to 2005 when one professional football team had beaten another team twice in the regular season before they met in the playoffs, that team had beaten the other in 10 of their playoff games.

(a) Use the binomial distribution to determine how surprising this result would be if the underlying probability of winning was 0.5 for each team.

(b) Based on this probability, do you consider the statistic cited (10 of 15 wins by the team that had won in the regular season) to be "significant"? Explain.

22. Wearing Helmets

A study conducted by the non-profit Safe Kids campaign in 2004 found that 33% of kids riding a bicycle on a residential street wore a safety helmet. Suppose that you watch a sample of 30 kids riding a bike on a residential street in your community. Let the random variable *H* represent the number of those 30 kids who are wearing a helmet.

(a) Would *H* follow a binomial distribution? Explain.

(b) If the rate of helmet wearing is the same in your community as in the Safe Kids study, how many kids would you expect to be wearing a helmet?

(c) Explain how you might use a six-sided die to simulate the distribution of *H*.

(d) Suppose that 15 of the 30 kids in your sample are wearing a helmet. Is that enough to convince you, at the 10% significance level, that the helmet wearing rate in your community is higher than 33%? Address this question with all of the steps of a test of significance.

23. Rock-Paper-Scissors (cont.)

Recall the "Rock-Paper-Scissors" scenario from Exercise 9. A friend conjectures that men are more likely to show rock than paper or scissors. To test this, you ask 50 male friends to play the game with you and you note the first choice of each male.

(a) State the appropriate hypotheses to be tested, in symbols and in words. (Pay particular attention to whether this is a one-sided or a two-sided test.)

(b) Let the random variable X represent the number of males that show "rock" as their first choice. What distribution does X follow if the null hypothesis is true? Explain.

(c) Suppose 23 men choose rock first. Determine the p-value from these data, and indicate whether the null hypothesis would be rejected at any commonly used significance level.

(d) Use technology to determine a 95% confidence interval for the parameter. Interpret this interval, including a clear statement of what the parameter is.

(e) Is the confidence interval consistent with your test result? Explain.

24. Water Oxygen Levels

Scientists often monitor the "health" of water systems to determine the impact of different changes in the environment. For example, Riggs (2002) reported on a case study that monitored the dissolved oxygen downstream from a commercial hog operation. There had been problems at this site for several years, including several fish deaths in the previous three years just downstream of a large swale through which runoff from the hog facility had escaped. The state pollution control agency wanted to see how often the dissolved oxygen level in the river was less than the 5.0 mg/l standard over the next three years. A measurement with a lower level was considered "non-compliant." Sampling was scheduled to commence in January of 2000 and run through December of 2002. The monitors took measurements at a single point in the river, approximately 6/10 of a mile from the swale, once every 11 days. Consistent with state water quality guidelines, the researchers decide that if it was determined that the river was non-compliant more than 10% of the time, then remedial action would be taken.

(a) Identify the observational units in this study.

(b) If the river is within state water quality guidelines, what does this imply about the value of π = long-run proportion of time the river is non-compliant?

(c) There were a total of 34 measurements taken in the first year, 19 of which were non-compliant. Does this sample provide strong evidence that π was above 0.10? Explain, using the binomial distribution to support your conclusion.

(d) Use technology to construction a 99% confidence interval for the population proportion of times the river is non-compliant. Include a one-sentence interpretation of this interval.

(e) Is 0.10 in your confidence interval? Explain why this result is consistent with your analysis in (c).

25. Left-handed Vegetarians

In a class of 36 statistics students, 6 reported that they were left-handed and 3 that they were vegetarians.

(a) Is this sample a random one from the population of all students at the college? Would you expect it to be representative of the population of all students at the college with regard to these variables?

For the remainder of this exercise, treat the sample as if it were a random one from the population.

(b) Conduct a binomial test of whether the population proportion of left-handers differs from 10%. First construct a bar graph to display the sample data, then state the hypotheses (in symbols and in words), and finally report the smallest significance level α at which you would reject this hypothesis. Also summarize your conclusion.

(c) Repeat (b) for testing whether the population proportion of vegetarians differs from 10%.

(d) With which variable do the sample data provide stronger evidence that the corresponding population parameter is not .10? Explain your answer.

26. Left-handed Vegetarians (cont.)

Reconsider the class results from the previous exercise.

(a) For each of the two variables (left-handedness and vegetarianism), determine a 95% binomial confidence interval for the population parameter.

(b) Determine the width of each interval.

(c) Because the sample size is the same for both variables, one might expect the two intervals to have the same widths. Do they? If not, which is wider? Explain why this makes sense.

27. Time Travelers
Reconsider the previous two exercises. The 36 students were also asked: "If time travel were possible, would you prefer to travel to the past or the future?" The past was preferred by 20 students and the future by 16.
(a) Does this sample provide much evidence to doubt that the percentage break-down in the population would be 50/50? Produce a bar graph, conduct a binomial test, and explain your conclusion.
(b) Determine and interpret a 95% binomial confidence interval for the population parameter. Also clearly identify in words what this parameter represents.
(c) If you define "success" to be "future" and another student defines success to be "past," how would your p-values for testing whether the population proportion differs from one-half compare? Explain.
(d) How would the competing definitions in (c) affect the confidence interval for the population proportion? Explain.

28. Detecting Cancer
In a study whose results were published in the *British Medical Journal* (Willis et al., 2004), researchers tested whether dogs' sense of smell can be helpful in detecting cancer in humans. Each dog used in the study was presented with seven urine samples to smell, with only one of the urine samples coming from a patient suffering from bladder cancer. Taken as a group, the dogs were tested on 54 trials, and they correctly identified the urine from the cancer patient in 22 trials. Conduct the appropriate test to assess whether these data are statistically significant. Be sure to state the appropriate hypotheses, check whether the binomial model applies, and calculate the p-value. Also summarize your conclusions and explain how they follow from your analysis.

29. Armrest Battles
In a study reported in 1982 by Kai, Khairullah, and Coulmas, researchers observed 426 pairs of passengers in "mixed-sex" seating arrangements on airplanes to see whether either person used the joint armrest. Observations were made after a beverage or meal was served. Passengers who were asleep or obviously couples were not counted. Data were collected by noting whether the man, the woman, both, or neither was using the armrest. Of the 426 pairs, the man used the armrest 284 times, the woman 57 times, both 37 times, and neither 48 times. We will consider only the 341 pairs for which one person or the other used the armrest.
(a) State the null and alternative hypotheses for testing whether men are more likely than women to use the shared armrest. (Be sure to define the parameter π in this context.)
(b) Use the binomial distribution to calculate the exact p-value of this test.
(c) Identify a potential confounding variable to explain the higher armrest use by males in this study.
(d) Calculate a 95% confidence interval for the probability that the shared armrest is used by the male. Include a one-sentence summary of this interval.
(e) Is 0.5 among the values in your confidence interval? Explain why this is consistent with your analysis in part (b).

30. Halloween Treat Choices (cont.)
Reconsider Investigation 1.9, in which you analyzed data from a study of 284 children's Halloween treat preferences. Let the parameter π represent the underlying probability that a child would choose the candy over the toy.
(a) Determine and interpret a 90% binomial confidence interval for π.
(b) Determine a 95% binomial confidence interval for π.
(c) Determine a 99% binomial confidence interval for π.
(d) How do these intervals compare to each other? Explain why this makes sense.
(e) Do some or all of these intervals include the value 0.5? Explain the relevance of this to the original research question in this study.

31. Halloween Treat Choices (cont.)
Reconsider the previous exercise.
(a) How would you expect those confidence intervals to change if you defined the parameter π to be the underlying probability that a child would choose the *toy* over the candy? Explain.
(b) Recalculate one of the previous intervals (90%, 95%, or 99%) with this new definition of the parameter π. Does the resulting interval validate what you expected in (a)? If not, explain how this interval compares to the previous one (with the other definition of π).

32. Relieving Back Pain
A study published in the journal *Neurology* (May 22, 2001) examined whether the drug botulinum toxin A is helpful for recurring pain among patients who suffer from chronic low back pain. The 31 subjects who participated in the study were randomly assigned to one of two treatment groups: 16 received a placebo of normal saline and the other 15 received the drug itself. The subjects' pain levels were evaluated at the beginning of the study and again after eight weeks. The researchers found that 2 of the 16 subjects who received the saline experienced a substantial reduction in pain, compared to 9 of the 15 subjects who received the actual drug.
(a) Use these data to conduct a two-sided test of whether one-third of all patients would experience substantial pain reduction from a placebo. Report the hypotheses and p-value, along with a summary of your conclusion. Also provide specific details about how the p-value is calculated.
(b) Repeat (a) for testing whether one-tenth of all patients would experience substantial pain reduction from a placebo.
(c) Determine a 95% confidence interval for estimating the proportion of all patients with this condition who would experience substantial pain reduction from a placebo.
(d) Does this interval include the value 0.333? Does it include the value 0.1? Explain why your answers make sense in light of the p-values in (a) and (b).
(e) Without doing any calculations, describe how you would expect your answers to (a), (b), and (c) to change if the study had involved 160 patients, 20 of whom experienced pain reduction. [*Hint*: Notice that the sample size is larger but the proportion of successes is unchanged.]

33. Hospital Mortality Rates (cont.)
Reconsider Investigation 1.3, in which you analyzed data from a sample of ten transplant operations performed by St. George's Hospital in London. Continue to consider testing whether the underlying mortality rate at the hospital exceeds the benchmark rate of 15%.
(a) Determine the smallest number of patient deaths (out of these ten operations) that would have been needed to produce a statistically significant result at the 5% level. Show the details of your calculations, or technology output, to support your answer.
(b) Repeat (a) for the 1% significance level.
(c) Repeat (a) and (b) for the larger sample of 361 patients whose records were analyzed.

34. Improved Batting Averages
Reconsider Investigation 1.6, in which you considered a baseball player who has historically had a 0.250 probability of success but now has a 0.333 probability. Continue to suppose that the player has a sample of 20 at-bats (trials) in which to convince his manager to reject the null hypothesis that $\pi = 0.250$ in favor of the alternative that $\pi > 0.250$. Now suppose that you want to choose the rejection region that minimizes the sum of the probabilities of Type I and Type II errors. Consider rejection regions of the form $\{X \geq k\}$, where X represents the number of hits (successes) in the sample of 20 at-bats (trials).
(a) For values of k from 0 to 20, calculate the probability of Type I error.
[*Hints*: It may be useful to keep in mind that $P(X \geq k) = 1 - P(X \leq k-1)$.]
(b) Produce a graph of these Type I error probabilities as a function of k.
(c) For values of k from 0 to 20, determine the probability of Type II error.
(d) Produce a graph of these Type II error probabilities as a function of k.
(e) Take the sum of the Type I and Type II error probabilities, and graph this sum as a function of k.
(f) What rejection region (value of k) minimizes the sum of these error probabilities?

35. Improved Batting Averages (cont.)
Reconsider the previous two exercises. Now suppose that you consider a Type I error to be twice as serious as a Type II error.
(a) Calculate and graph $2 \times P(\text{Type I error}) + P(\text{Type II error})$, as a function of k.
(b) What rejection region (value of k) minimizes the sum of these error probabilities?
(c) Repeat, now supposing that you consider a Type I error to be four times as serious as a Type II error.
(d) Do you notice a pattern in how these rejection regions change, as you consider Type I error to be increasingly more important than a Type II error? Explain why this makes sense.

36. Improved Batting Averages (cont.)
Reconsider the previous exercises. Suppose that the player has a sample of n at-bats (trials) in which to convince his manager to reject the null hypothesis that $\pi = 0.250$ in favor of the alternative that $\pi > 0.250$, using the $\alpha = 0.05$ significance level.
(a) Intuitively, without doing any calculations but perhaps sketching some graphs, do you expect the power of this test to be an increasing function of n, a decreasing function of n, or neither? Explain.
(b) Determine how large the sample size n needs to be, in order for the power to be at least 0.75. Explain the details of your calculation, but feel free to use technology to check your answer.
(c) Now suppose that the player's success probability is denoted by π_a, not necessarily equal to 0.333. For a fixed sample size n and significance level α, do you expect the power of this test to be an increasing function of π_a, a decreasing function of π_a, or neither? Explain.
(d) With a sample size of $n = 50$ and the $\alpha = 0.05$ significance level, determine how large π_a has to be in order for the power to be at least 0.75. Explain the details of your calculation, but feel free to use technology to check your work.

37. Halloween Treat Choices (cont.)
Reconsider Investigation 1.9, in which you analyzed data from a study of 284 children's Halloween treat preferences. Let the parameter π represent the underlying probability that a child would choose the candy over the toy.
(a) State an appropriate null and alternative hypothesis involving this parameter (in symbols and in words), for testing whether there is strong evidence of a preference for either the toys or the candy.
(b) Explain what a Type I error would represent in this context.
(c) Explain what a Type II error would represent in this context.

(d) Suppose 60% of trick-or-treaters prefer the toy. Explain what "power" would represent in this context (loosely outline how you would determine the power of the test, you do not need to perform the calculations).
(e) Explain the effect of increasing the sample size on the power of this test.
(f) Now suppose that 55% of all children prefer candy over a toy as a Halloween treat. Determine the power of the test that you performed, using the 0.05 level of significance.
(g) How does the power in (f) compare to that in (d)? Explain why this makes sense.

38. Cola Discrimination?
A teacher doubted whether his students could distinguish between two different brands of cola soft drink (say, Coke and Pepsi). He presented each of his 48 students with three cups of cola. Two contained the same brand, and the third contained the other brand. Each student was asked to identify the cup containing cola that differed from the other two cups. Let π represent the probability that a student can correctly identify the "odd" brand. The hypotheses to be tested are $H_0: \pi = 1/3$ vs. $H_a: \pi > 1/3$.
(a) Describe (in words) what Type I error means in this situation.
(b) Describe (in words) what Type II error means in this situation.
(c) Describe (in words) what power means in this situation.
For the remaining questions, you may use either the Power Simulation applet for an approximate answer or R/Minitab for an exact answer. (Include screen captures of applet results or R/Minitab output with your answers.)
(d) Determine the rejection region for this test, using the $\alpha = 0.05$ significance level.
(e) Calculate the power of this test, using the $\alpha = 0.05$ significance level, when the success probability is actually $\pi = 0.5$. Also be sure to write this probability as P(X ___ k), where you indicate the appropriate probability distribution of X, fill in the blank with the appropriate inequality, and indicate the appropriate value of k.)
(f) Conjecture how the power would change if the success probability were larger. Explain why this makes sense intuitively. Then calculate the power when $\pi = 2/3$, and comment on whether this supports your answer.
(g) Conjecture how the power would change if the significance level were smaller. Explain why this makes sense intuitively. Then calculate the power using $\alpha = 0.01$ (for an alternative value of $\pi = 0.5$), and comment on whether this supports your answer.
(h) Conjecture how the power would change if the sample size were larger. Explain why this makes sense intuitively. Then calculate the power using $n = 96$ (with $\alpha = 0.05$ for an alternative value of $\pi = 0.5$), and comment on whether this supports your answer.

39. Water Oxygen Levels (cont.)
Recall the dissolved oxygen study (Exercise 24) with 34 observations. The investigators decided they wanted to be able to detect a non-attainment rate of 25% and that they wanted the Type I and Type II error rates, considering this alternative value of 25%, to be reasonably similar.
(a) Identify what decisions would be represented by a Type I error and by a Type II error in this context. Also, describe possible consequences from each type of error.
(b) Suppose the quality assessment manager states that the probability of a Type I error can be at most 0.15. What is the "cutoff" value for the rejection region? That is, find the smallest x so that $P(X \geq x) \leq 0.15$ when $\pi = 0.10$.
(c) Was the observed sample result (19 out of 34) in this rejection region?
(d) What is the probability of a Type II error for the cutoff value in (b) and the alternative of $\pi_a = 0.25$? Does this false negative rate appear reasonable, considering the investigator's view that the two types of errors are equally serious?

(e) How would the error rates in (b) and (d) change if instead we made the cutoff 5 or more? Explain intuitively, and then confirm your answer with appropriate calculations.
(f) Would the cutoff suggested in (e) be appropriate if the investigators considered a Type I error to be more serious than a Type II error, or vice versa? Explain.

40. The Monty Hall Problem
In the famous "Monty Hall problem," a game show contestant is asked to choose one of three doors, behind only one of which is a prize. Then the host opens one of the other doors, one which he knows does not have the prize, and asks the contestant if she would then prefer to switch to the remaining door. Suppose that you want to play a series of games with the switching strategy to investigate whether the probability of winning with that strategy is higher than 1/3.
(a) State the null and alternative hypotheses, in symbols and in words.
(b) Suppose that you play this game for a sample of 25 trials. Determine the rejection region corresponding to the $\alpha = 0.10$ significance level.
It can be shown that the probability of winning the prize doubles from 1/3 to 2/3 by switching doors.
(c) Determine the probability of committing a Type II error in this study.
(d) Repeat (b) and (c) for the $\alpha = 0.05$ significance level.
(e) Repeat (b) and (c) for the $\alpha = 0.01$ significance level.
(f) Describe how the probability of Type II error changes as the significance level α decreases. Explain why this makes intuitive sense.

41. Heart Transplant Mortality (cont.)
Reconsider the previous exercise. Continue to consider testing whether the data suggest that the underlying mortality rate of heart transplant procedures at this hospital exceeded 0.15, based on a sample of 361 patients.
(a) Determine the rejection region, using the $\alpha = 0.05$ significance level.
(b) Determine the probability of Type II error, still using the $\alpha = 0.05$ significance level, if the underlying mortality rate actually equals 0.20.
(c) Repeat (a) and (b) using the $\alpha = 0.01$ significance level. Describe how the rejection region and Type II error probability change between the two values of α, and explain why these changes make intuitive sense.

42. Compounding Errors
Suppose that you conduct 10 independent tests of significance, using the $\alpha = 0.05$ significance level for each test. Also suppose that, unknown to you, the null hypothesis is actually true for every test. Let the random variable R be the number of tests for which you (mistakenly) reject the null hypothesis.
(a) Does rejecting the null hypothesis constitute a Type I or a Type II error in this situation?
(b) Explain why the random variable R can be considered as having a binomial distribution, and specify its values of n and π.
(c) Determine the expected value of R, the number of tests for which you (mistakenly) reject the null hypothesis.
(d) Determine the probability that you mistakenly reject the null hypothesis for at least one of these tests (i.e., that $P(R > 0)$).
(e) Repeat (c) and (d) if you conduct 20 independent tests of significance, using the $\alpha = 0.05$ significance level for each test, where the null hypothesis is actually true for every test.
(f) Repeat (e), supposing that you use the $\alpha = 0.10$ significance level for each test.
This exercise illustrates a concern with conducting a large number of tests of significance.

43. Compounding Errors (cont.)

Reconsider the previous exercise. Let n represent the number of independent tests to be performed, and let α represent the significance level. Continue to assume that, unknown to you, the null hypothesis is actually true for every test. Continue to let the random variable R be the number of tests for which you (mistakenly) reject the null hypothesis.

(a) Determine the expected value of R, as a function of n and π.
(b) Is this expected value an increasing or a decreasing function of n? Explain why your answer makes intuitive sense.
(c) Is this expected value an increasing or a decreasing function of α? Explain why your answer makes intuitive sense.
(d) Determine the probability that you mistakenly reject the null hypothesis for at least one of these tests (i.e., that P($R > 0$)), as a function of n and α.
(e) Is this probability an increasing or a decreasing function of n? Explain why your answer makes intuitive sense.
(f) Is this probability an increasing or a decreasing function of α? Explain why your answer makes intuitive sense.
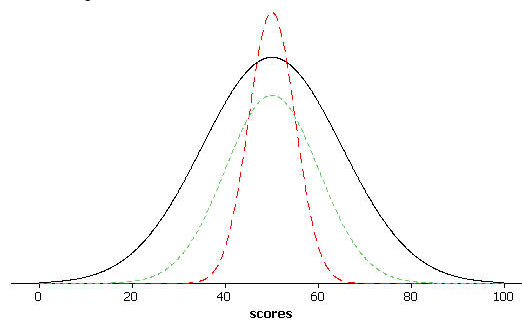
44. Normal Curves

Consider the following four normal curves modeling exam scores for four classes:
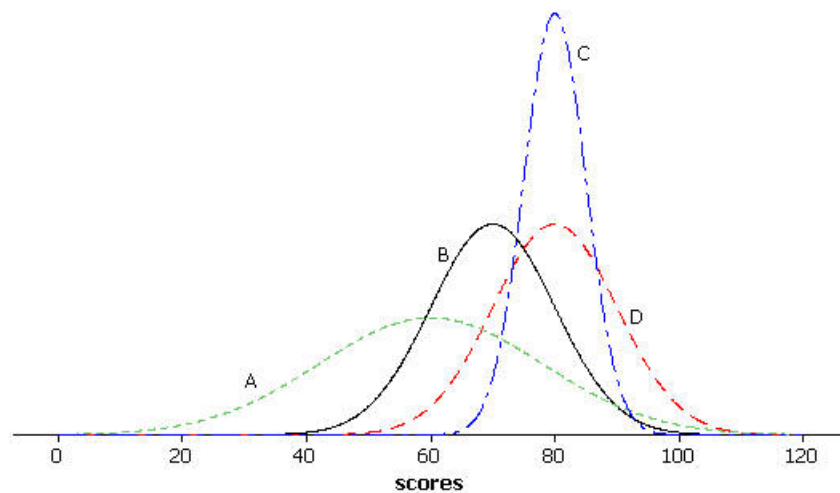
Approximate (as well as you can from the graphs) the mean and standard deviation of the exam score model for each class.

45. Bell-shaped Curves

Consider the following three bell-shaped curves:

Explain why these cannot all be normal curves.

46. Competitive Advantage of Uniform Color? (cont.)
For the Competitive Advantage of Uniform Color data (Exercise 11), assuming $\pi = 0.5$:
(a) Is the sample size in this study large enough for the Central Limit Theorem for a sample proportion to apply?
(b) Assuming the CLT applies, specify the shape, mean, and standard deviation predicted by the CLT for the distribution of sample proportions. Include a well-labeled "sketch" (by hand or computer rendered) of this distribution. [*Hint*: By well-labeled, we mean a horizontal axis label and indication of scaling on the horizontal axis showing the mean and one standard deviation on each side of the mean.]
(c) Compare the predicted distribution with simulation results from either the Reese's Pieces applet or the One Proportion Inference applet (comment on each of shape, mean, and SD). Be sure to include your output.
(d) Use technology to estimate the probability of obtaining a sample proportion of at least 0.543 using the normal approximation. (Be very clear how you have done so and include your output.) How does this compare to the p-value you found using the binomial distribution?

47. Rock n' Roll (cont.)
Reconsider Exercise 4. Suppose that we obtain a list of all songs with a color in the title, and suppose that we take a random sample of 75 songs from that list and determine the sample proportion, $\hat{p}$, of songs that are about the color blue. Suppose for now that the claim (30% of all songs with a color in the title are about the color blue) is true.
(a) Verify that the CLT for a sample proportion applies here.
(b) What does the CLT say about the sampling distribution of $\hat{p}$? (Mention shape, center, and spread, and also draw a well-labeled sketch of the sampling distribution.)
(c) Use the CLT to approximate the probability that between 24% and 36% of the songs in the sample will be "blue." Include a one-sentence summary of what the calculated probability signifies.
(d) Use the binomial distribution to calculate the probability in (c) exactly. [*Hint:* First use the sample size of 75 to convert the 24% and 36% into counts.]
(e) How close did the normal approximation in (c) come to the exact binomial probability in (d)?
(f) Use the continuity correction to improve on the normal approximation in (c).
(g) Repeat (c)–(f) for finding the probability that between 20% and 40% of the songs in the sample will be "blue."

48. Rock n' Roll (cont.)
Reconsider again the claim that 30% of all songs with a color in the title are about the color blue. Again suppose that we obtain a list of all songs with a color in the title, and suppose we take a random sample of 75 songs from that list and determine the sample proportion, $\hat{p}$, of songs that are about the color blue.
(a) Using the CLT, determine the values of $\hat{p}$ that would lead to rejecting the null hypothesis at 30% of all songs with a color in the title are about the color blue at the 5% level of significance.
(b) Suppose that actually 40% of the population of all songs with a color in the title are about the color blue. Sketch a graph of the resulting sampling distribution, using 40% as the value for π, and using the values you determined in (a), determine the probability that we would fail to reject the null hypothesis of 30% at the 5% level of significance. Is this a Type I or a Type II error?

49. Which Tire? (cont.)
Reconsider Exercise 7. Here are results from a different class:

Left front	Left rear	Right front	Right rear
11	5	19	2

(a) What proportion of students picked the right front tire? Is this in the direction of the research conjecture?
(b) Would it be valid to apply the one-proportion z-test with these data? Explain.
(c) Regardless of your answer to (b), calculate the one-proportion z-test using technology. Include your output and report the test statistic and p-value (both with and without a continuity correction). [*Hint*: To do the continuity correction in Minitab or Normal Probability Calculator applet, go back to the normal distribution, using the mean and SD specified by the CLT.]
(d) Write a one-sentence interpretation of the test statistic in this context.
(e) What *test decision* (reject or fail to reject the null hypothesis) would you make based on this p-value?
(f) Regardless of your answer to (b), calculate and interpret a 95% z-confidence interval for the parameter of interest.
(g) Explain what is meant by the phrase "95% confidence" in this context.
(h) Reconsider the class of 54 students, in which 24 picked the right front tire. *Without carrying out any new inference procedure calculations*, explain how the test statistic, p-value, and confidence interval for the 54 student class to compare to the results you found here. Justify your answers.

50. Magical Numbers
Think of a number satisfying these conditions:
- The number has two digits.
- Both digits are odd.
- The two digits are different from each other.

(a) Report the number that you think of.
(b) How many numbers satisfy these conditions?
A magician on television claimed that the number 37 is somehow special and so people tend to pick it more often than would be expected by chance.
(c) If the numbers satisfying these conditions are all equally likely, what is the probability that a person would pick the number 37?
(d) State the null and alternative hypotheses for testing the magician's claim.
(e) Describe what Type I and Type II errors signify in this situation.
(f) With a sample of 25 people, how many would have to pick 37 in order for the result to be statistically significant at the 0.05 level?
(g) Suppose that the probability of picking 37 is actually 0.10, twice as high as equal likeliness would suggest. Determine the probability that a sample of 25 people would produce a result in the rejection region of (f). Write a sentence or two interpreting what this probability reveals.
(h) What term describes the probability that you calculated in (g) – *significance, confidence, power*, or *influence*?

51. Magical Numbers (cont.)
Previous studies have investigated the question of whether people tend to think of an odd number when they are asked to think of a single-digit number (0 through 9). Combining results from several studies, Kubovy and Psotka (1976) used a sample size of 1770 people, of whom 741 thought of an even number and 1029 thought of an odd number.
(a) Use the normal approximation (Wald procedure) to determine a 95% confidence interval for the relevant parameter. Show how to calculate this interval by hand.

(b) Interpret what this confidence interval in (a) says. [*Hint*: You are 95% confident of what?]
(c) Determine a 99% confidence interval and a 99.9% confidence interval for the relevant parameter based on the sample data. (Feel free to use technology.)
(d) Comment on how the three confidence intervals compare to each other. [*Hint*: Be sure to comment on both width and midpoint.]
(e) Based on these confidence intervals, would you say that the sample data provide strong evidence that people have a tendency to think of an odd number rather than an even number? Explain how you are deciding.
(f) Suppose that you want to estimate the relevant parameter to within ± 0.02 with 99% confidence. Determine how many people you would have to ask. [*Hint*: Use the result of the previous studies in determining the sample size.]
(g) Re-answer (f) without using the result of the previous studies, so using 0.5 as an estimate for the observed proportion. Comment on how this changes the necessary sample size.
The researchers found that 503 of the 1770 people thought of the number 7.
(h) Determine a 95% confidence interval for the long-run probability that a person thinks of the number 7. Does this confidence interval suggest that people have a tendency to think of the number 7 more often than by random chance? Explain.

52. Renaming Brides
How often do brides keep their own last name, or adopt their husband's last name, or use a combination of both names? Researchers investigated these questions by examining a random sample of 600 heterosexual wedding announcements appearing in *The New York Times* between the years 2001 and 2005 (Kopelman et. al., 2009). They found that 18% of the 600 brides kept their own last name.
(a) Check the sample size condition(s) for whether a one-proportion *z*-interval can be used to estimate the population proportion of brides between 2001 and 2005 who kept their own last name.
(b) Calculate a 95% confidence (one-proportion z-) interval for the population proportion of brides between 2001 and 2005 who kept their own last name.
(c) Interpret the confidence interval that you determined in (b).
(d) To what population would you feel comfortable generalizing this confidence interval: all brides in the world between 2001 and 2005, or all brides in the United States, or …? Justify your response.
(e) Determine the sample size needed to estimate the population proportion of brides between 2001 and 2005 who kept their own last name to within ± 0.03 with 95% confidence. Show the details of your calculation.
The Kopelman et. al. researchers reported that in addition to the 18% of the 600 brides who kept their own last names, another 10% of the 600 brides kept their maiden name in a modified form, for example by using her maiden name as her middle name or by hyphenating her maiden name with her husband's name. Moreover, the researchers reported that 45% of the 600 brides adopted their husband's last name, and no information about names could be determined for the remaining 27% of the 600 brides.
Now eliminate from the sample the 27% of the 600 brides for whom name information could not be determined, and combine those who kept their own last name and those who kept their maiden name in modified form into one group.
(f) What proportion of the new sample (those brides for whom name information could be determined) kept their last name in full or modified form?
(g) Use your answer to (f) to produce a 95% confidence interval for the relevant parameter. Also interpret this interval.

53. Baseball Big Bang?

A reader wrote in to the "Ask Marilyn" column in *Parade* magazine to say that his grandfather told him that in 3/4 of all baseball games, the winning team scores more runs in one inning than the losing team scores in the entire game. (This phenomenon is known as a "big bang.") Marilyn responded that this probability seemed to be too high to be believable. Let π denote the actual probability that a Major League Baseball game results in a "big bang."

(a) Restate the grandfather's assertion as a null hypothesis, in symbols and in words.
(b) Report Marilyn's response as an alternative hypothesis, in symbols and in words.

To investigate this claim, one of the authors examined the 45 Major League baseball games played on September 17 – 19, 2010, and found that 21 of these 45 games contained a big bang.

(c) Calculate the sample proportion of games that had a big bang, and denote it with the appropriate symbol.
(d) If the grandfather's claim is true, how many standard deviations below the mean is the observed sample proportion? Also denote this with the appropriate symbol.
(e) Use the normal distribution to determine the approximate p-value, first without using the continuity correction and then with using the continuity correction. Also produce (and submit) an appropriately labeled and shaded graph for each of these normal calculations.
(f) Would you conclude that the sample data provide strong evidence to support Marilyn's contention that the proportion cited by the grandfather is too high to be the actual value? Explain your reasoning, as if writing to the grandfather, who has never taken a statistics course.
(g) Marilyn went on to assert that she believes the actual probability of a big bang to be 0.5. Conduct a two-sided test of this hypothesis. Report the hypotheses, test statistic, and p-value. Again perform the calculations with and without using the continuity correction. Also calculate the exact p-value from the binomial distribution. Produce (and submit) appropriately labeled shaded graphs for all of these calculations. Comment on whether the continuity correction is helpful here. State the test decision at the $\alpha = .10$ significance level, and summarize your conclusion.

54. Competitive Advantage from Uniform Color? (cont.)

Recall the study of 457 matches in four combat sports (boxing, tae kwon do, Greco-Roman wrestling, freestyle wrestling) at the 2004 Olympic Games. Competitors in these sports were randomly assigned to wear either a red or a blue uniform. The researchers found that the competitor wearing red defeated the competitor wearing blue in 248 matches, and the competitor wearing blue emerged as the winner in 209 matches.

(a) Identify the observational units and variable in this study.
(b) Verify the conditions for using the Wald (*z*-) procedure to determine a 95% confidence interval for the probability that the competitor wearing red wins a match.
(c) Calculate this 95% confidence interval.
(d) Interpret what this interval reveals: We are 95% confident that …
(e) Interpret what the 95% confidence *level* means in this context.
(f) Repeat (c) for a 99% confidence interval.
(g) Describe how these two intervals compare, in terms of both their midpoints and widths.
(h) Do these intervals suggest that one uniform color or the other provides a competitive advantage? Explain.
(i) Suppose that the sample size had been four times larger, and the sample proportion had been identical to the actual study. Determine a 95% confidence interval in this case, and comment on how it compares to the interval in (c). [*Hint*: Be as specific as possible, and be sure to comment on both midpoint and width.]
(j) Determine how large a sample size would be necessary to estimate the actual probability to within 5 percentage points with 90% confidence.

55. Political Jokes?
The University of Pennsylvania's National Annenberg Election Survey of 2004 studied the humor of late-night comedians. They performed a content analysis of the jokes made by Jon Stewart during the "headlines" segment of *The Daily Show* from July 15 – September 16, 2004. They found that 83 of the 252 jokes were of a political nature.
(a) Identify the observational units and the variable in this study.
(b) Produce a graphical and numerical summary, along with a one-sentence description, of this sample.
(c) Consider these 252 jokes to be a random sample from the joke-producing process for this show. Check technical conditions, and use these sample data to produce 90%, 95%, and 99% confidence intervals for the underlying probability that a joke is political in nature.
(d) How do the widths of these confidence intervals compare? Explain why that makes sense.
(e) How do the midpoints of these confidence intervals compare? Explain why that makes sense.
(f) Based on these confidence intervals, what can you say about the p-value for testing whether half of *The Daily Show* jokes are political in nature? Explain.
(g) Give a value of π that would be rejected at the 5% level but not at the 1% level. Explain.

56. Volunteerism
In September of 2003 the Bureau of Labor Statistics conducted a study of volunteerism in the United States as part of its ongoing Current Population Survey. A random sample of about 60,000 people (age 16 and over) was asked whether they had participated in any volunteer activities through or for an organization in the past year. It turned out that 28.8% said that they had participated in volunteer activities.
(a) Produce a graphical and numerical summary, with a one-sentence description, about this sample.
(b) Use these data to conduct a one proportion *z*-test of the hypothesis that 30% of adult Americans participated in volunteer activities that year, against the alternative that this percentage was not 30%. State the hypotheses in symbols and words, check the technical conditions, calculate the test statistic and p-value, and indicate whether the result is statistically significant at the 0.01 level. Include a well-labeled sketch of the sampling distribution and indicate the area represented by the p-value. Also summarize your conclusion about the population and explain how it follows from your test.
(c) Follow up on this test by producing a 99% confidence interval for the population proportion who participated in volunteer activities that year. Include a one-sentence interpretation of this interval.
(d) Is the value 0.3 contained within the confidence interval? Is this consistent with your test decision? Explain.
(e) Explain the difference in the following statements. Also indicate which is the appropriate conclusion to draw from the data in this particular study, and explain your choice.
   • The sample data provide strong evidence that the proportion of adult Americans who participated in volunteer activities was very different from 30%.
   • The sample data provide very strong evidence that the proportion of adult Americans who participated in volunteer activities was different from 30%.

57. Halloween Treat Choices (cont.)
Inspired by the Halloween treat study described in Investigation 1-9, a pair of students (Avery & Botts, 2004) conducted their own study of whether trick-or-treaters would choose fruit-flavored candy (high in sugar) or chocolate candy (high in both sugar and fat). The students gave justification for why they felt the sample they obtained was representative of all trick-or-treaters in their community. Prior to conducting the study, the students wanted to test whether half of children would choose the chocolate candy, while some classmates conjectured that two-thirds would choose the chocolate candy, and others proposed that three-fourths would choose the chocolate candy.

(a) State the null and alternative hypotheses, in words and in symbols, for testing the students' suspicion that half of children would choose chocolate candy.

The students found that of 127 of 191 children in the study chose the chocolate candy.

(b) Calculate the proportion of children in the study who chose the chocolate candy. Is this a parameter or a statistic? What symbol do we use to denote it?

(c) Verify that the technical conditions for the one proportion z-test are satisfied in this study.

(d) Calculate the z-test statistic and p-value, and include a well-labeled sketch of the sampling distribution denoting the area represented by the p-value. Would you reject the null hypothesis at the 0.10 level? What about the 0.05 and 0.01 levels? What is the smallest level of significance for which you would reject the null hypothesis here?

(e) Conduct significance tests for the other two hypotheses suggested above. Report the hypotheses, technical condition checks, test statistics, and p-values. Summarize your conclusions.

(f) After verifying the appropriate technical conditions, determine and interpret a 90% confidence interval for the population proportion who would choose the chocolate candy. Is this interval consistent with the p-values in (d)? Explain.

(g) Without knowing more about the study, to what population would you be willing to generalize your findings?

58. Halloween Treat Choices (cont.)

Reconsider the previous question.

(a) If two-thirds of the trick-or-treaters in this community prefer the chocolate candy, what is the power of the test at the 5% level of significance? (Include relevant sketches.)

(b) Repeat (a) if three-fifths of the trick-or-treaters in this community prefer the chocolate candy.

Now consider how details of your analysis would change if the parameter of interest was the proportion of children who would choose the fruit candy, rather than the proportion who would choose the chocolate candy.

(c) How would this affect (if at all) your confidence interval calculation and interpretation? Explain.

(d) How would this affect (if at all) your p-value calculation and test conclusion for testing the value .5 against the two-sided alternative? Explain.

59. Sarah the Chimpanzee

Early research has found chimpanzees able to solve complex problems, like fitting sticks together to make a rake to gather food. In a 1978 study published in *Science*, Premack and Woodruff asked "To what extent does the chimpanzee comprehend the elements of a problem situation and potential solutions?" An adult chimpanzee (Sarah) was shown 30-second videotapes of a human actor struggling with one of several problems (for example, not able to reach bananas hanging from the ceiling, a record player not playing). Then Sarah was shown two photographs, one that depicted a solution to the problem (like stepping onto a box, plugging in the record player) and one that did not match that scenario. On seven of eight problems, the animal consistently chose the correct photograph.

(a) Identify the observational units and variable in this study.

(b) State the null and alternative hypotheses for testing whether Sarah is more likely to pick the correct photograph than if she was just randomly guessing between the two solutions each time.

(c) Is it reasonable to model this study as a binomial process? Explain.

(d) Determine the p-value for Sarah and provide a detailed interpretation of the p-value you find.

(e) Summarize the conclusions you would draw from this study. Do you think Sarah got lucky or do you think something other than random chance was at play? How strong is the evidence?

60. A Penny for Your Thoughts (cont.)
In June of 2004, the Harris organization asked a random sample of 2136 adult Americans: "Would you favor or oppose abolishing the penny so that the nickel would be the lowest denomination coin?" It turned out that 59% of those adults answered "oppose."
(a) Identify the observational units and variable in this study.
(b) Identify the statistic and parameter in this study.
(c) Check whether the technical conditions for the Wald confidence interval procedure are satisfied in this study.
(d) Determine and interpret a 95% confidence interval for the population proportion who oppose abolishing the penny. (Use the Wald interval if appropriate, the adjusted Wald procedure if not.)
(e) Does this interval suggest that more than half of all American adults oppose abolishing the penny? Explain.
(f) If the study had involved a random sample of 1136 adult Americans, and all else had turned out the same, would the width of this confidence interval change? If so, how? Explain briefly.
(g) If the study had involved a random sample of 1136 adult Americans, and all else had turned out the same, would the midpoint of this confidence interval change? If so, how? Explain briefly.
(h) If another random sample of 2136 adult Americans were asked this question, is the probability 0.95 that the sample proportion who answer "oppose" would fall within the interval from (d)? Explain briefly.

61. Detecting Cancer (cont.)
Recall from the Exercise 28 the study on whether dogs' sense of smell can be helpful in detecting cancer in humans. Each dog used in the study was presented with seven urine samples to smell, with only one of the urine samples coming from a patient suffering from bladder cancer. Taken as a group, the dogs were tested on 54 trials, and they correctly identified the urine from the cancer patient in 22 trials. Even though the technical conditions for the one proportion z-test are not quite satisfied here, we can still calculate a z-statistic, which will tell us how many standard deviations the observed results falls from what the null hypothesis would predict. But we should not proceed to calculate a p-value from the normal distribution, because the normal approximation to the binomial distribution is not valid here.
(a) State the null and alternative hypotheses for testing whether the dogs are able to correctly identify the urine from the cancer patient more often than random chance would predict.
(b) Show that the technical conditions for the one proportion z-test are not quite satisfied here.
(c) Determine how large the sample size would need to be in order for calculation of the p-value of the one proportion z-test to be appropriate here.
(d) Check technical conditions, and then use the Wald or adjusted Wald procedure to produce and 95% confidence interval for the population proportion of correct identifications. As always, include a one-sentence interpretation of the interval.
(e) Comment on what these intervals reveal about the dogs' effectiveness at detecting urine cancer.

62. Pilot Study (cont.)
Recall from Exercises 16 and 17 that we encountered one female in a sample of eight pilots on a recent trip.
(a) Are the technical conditions for the Wald confidence interval procedure satisfied here?
(b) Use the adjusted Wald procedure to estimate the population proportion of all commercial pilots who are women, based on these data.

(c) Using this sample as a pilot study (pardon the pun), determine how large a sample would be needed to estimate the population proportion to within $\pm$ 0.075 with 90% confidence. [*Hint*: Remember to report n, not $n + 4$.]

63. Random Babies
Recall the scenario from Investigation 0: Suppose that on one night at a certain hospital, four mothers give birth to baby boys. As a very sick joke, the hospital staff decides to return babies to their mothers completely at random. In this situation, the theoretical probability of obtaining zero matches is 9/24, or 0.375. For now let π denote this value, so $\pi = 0.375$. Suppose that you conduct 1000 repetitions of simulating this process of returning babies to their mothers at random, so each repetition has probability 0.375 of resulting in zero matches. Let $\hat{p}$ denote the proportion of your 1000 repetitions that result in zero matches.

(a) Describe what the Central Limit Theorem says about the sampling distribution of $\hat{p}$ if you were to repeatedly conduct simulations of 1000 repetitions each. (Comment on shape, center, and spread, and draw a well-labeled sketch of the sampling distribution.)

(b) Between what two values will $\hat{p}$ fall, with probability 0.05? [*Hint*: Go 1.96 standard deviations on either side of π.]

(c) If you want the simulation to approximate π to within $\pm$ 0.01 with 99% confidence, how many repetitions are necessary?

(d) Now let π represent the theoretical probability that all four mothers get the correct baby. Then $\pi = 1/24$, but suppose that you want the simulation to approximate π to within $\pm$ 0.005 with 99% confidence. Determine how many repetitions of the simulation would be required.

The previous exercise shows that you can use the Central Limit Theorem to determine how many repetitions of a simulation are necessary to estimate the underlying theoretical probability with a desired level of accuracy and precision.

64. Simulating Wald Intervals
One of the criticisms against the Wald method is that its performance can be quite haphazard, meaning that it doesn't always improve as n increases with a fixed value of π, and it doesn't always improve as π gets closer to 0.5 for a fixed value of n. You will investigate this claim using the Simulating Confidence Intervals applet. Start with the population proportion $\pi = 0.5$ and the sample size $n = 14$.

(a) Is the sample size condition for the validity of the Wald method satisfied here?

(b) Simulate the drawing of 200 random samples, and the construction of 95% confidence intervals for π, in this situation. Then keep clicking on "sample" until you have produced at least 5000 samples and their resulting intervals. What percentage of the nominal 95% confidence intervals succeed in capturing the actual value of π?

(c) Is this percentage reasonably close to 95%? How well would you say the Wald procedure performs in this situation?

(d) Repeat (a)–(c) with the population proportion $\pi = 0.5$ and the sample size $n = 40$.

(e) In which of these situations ($n = 14$ or $n = 40$) should the Wald method have performed better? Why? Is that how it turned out?

(f) Repeat this analysis by comparing the performance of the Wald procedure when $n = 100$ and $\pi = 0.106$ to its performance when $n = 100$ and $\pi = 0.107$. Summarize your findings.

(g) Apply the adjusted Wald procedure in all four of these situations, and comment on its performance relative to that of the Wald procedure. Does its performance appear to improve as n increases?

65. Backpack Weights

A growing problem in American schools involves students who develop back problems, possibly as a result of carrying too much weight in their backpacks. It has been proposed that students should carry no more than 10% of their body weight in their backpack. To investigate how much students carry in their backpacks, student researchers at Cal Poly sampled 100 students. They asked these students to report their body weight, and they weighed how much was carried in their backpack. These data, along with the ratio of backpack weight to body weight, are in the file `backpack.txt`.

(a) Use technology to create a new variable: whether or not the student carries at least 10% of his/her body weight in his/her backpack. Is this a quantitative or a categorical variable?

(b) Examine graphical and numerical summaries of this variable, and comment on its distribution in this sample.

(c) Conduct a significance test of whether the sample data suggest that the less than half of all Cal Poly students carry at least 10% of their body weight in their backpack. Report hypotheses, comment on technical conditions, and calculate the test statistic and p-value. Include a well-labeled sketch of the sampling distribution for the test statistic and indicate the area represented by the p-value. Also summarize your conclusion and explain how it follows from your test.

(d) Construct and interpret a 90% confidence interval for the population proportion who carry at least 10% of their body weight in their backpack. [*Hint*: Be sure to check whether the technical conditions for the Wald procedure are satisfied; use the adjusted Wald procedure if not.]

(e) Construct and interpret a 95% confidence interval for the population proportion who carry at least 15% of their body weight in their backpack.

66. Hound of the Baskervilles

Phillips et al. (2001) report on a "Hound of the Baskervilles" effect, in which they conjecture that Chinese- and Japanese-Americans, perhaps frightened by the negative connotations of the number 3 in their culture, die more often on the 3rd day of a month than on the 2nd or 4th day of the month. They analyzed death records of Chinese- and Japanese-Americans from California for the years 1989–1998, focusing on what they termed "chronic," inpatient, coronary deaths. They found 404 deaths occurring on days 3, 4, or 5 of a month, with 157 of those deaths occurring on day 3.

(a) What proportion of these 404 deaths occurred on day 3? Is this higher than 1/3? Produce a bar graph to display these data.

(b) If in fact 1/3 of all deaths on day 3, 4, or 5 occur on day 3, what is the probability that a sample of 404 such deaths would reveal 157 or more on day 3?

(c) Is this probability small enough to suggest that Chinese- and Japanese-Americans do die on the 3rd day of the month more often than would be expected by chance? Explain.

Smith (2002) conducted a follow-up study in which he broadened the types of deaths to include all inpatient coronary deaths and all coronary deaths for the same years. He also studied deaths in the subsequent three years 1999-2001. Some of his findings are in the table:

Types of death	Years	# Deaths on day 3, 4, 5	Proportion of deaths on day 3
All coronary	1989–1998	1391	33.9%
Inpatient coronary	1989–1998	540	36.5%
Inpatient, chronic coronary	1999–2001	133	34.6%

(d) For each of these three categories of deaths, determine the p-value for a binomial test of whether the underlying death rate exceeds 1/3.

(e) Based on these p-values, do Smith's data seem to support the contention that Chinese- and Japanese-Americans die more often on the 3rd day of the month than on the 2nd or 4th day? Explain.

67. Left-Handed Vegetarians (cont.)
Reconsider Exercises 25 and 26 which reported the results of a class survey on three questions:
- Of the 36 students, 3 were vegetarian.
- Of the 36 students, 6 were left-handed.
- Of the 36 students, 20 preferred to travel to the past and 16 to the future.

(a) For each of these three questions, estimate the relevant population proportion (again treat these as if they were random samples from the population) with a 95% Wald confidence interval, a 95% adjusted Wald confidence interval, and a 95% binomial confidence interval.
(b) Comment on how similar the three types of intervals are. For which question are they most similar? For which question are they most different?

68. *Swain v. Alabama*
In *Swain v. Alabama* (1965) it was alleged that there was discrimination against blacks in grand jury selection. Swain, a black man, was convicted in Talladega County, Alabama, of raping a white woman and was sentenced to death. At that time in Alabama, only men over the age of 21 were eligible for jury duty. Census data suggested that about 26% of those eligible for grand jury service were black, yet a "random sample" of 1050 individuals called to appear for possible grand jury duty yielded only 177 blacks.
(a) Produce numerical and graphical summaries of the sample results.
(b) Define the parameter of interest and indicate the symbol used to represent it.
(c) State the null and alternative hypotheses.
(d) Describe what a Type I and a Type II error would represent in this situation. What are the consequences of each type of error?
(e) Is the normal model valid here? If so, use the CLT to calculate the test statistic and p-value (include a sketch). If not, calculate the p-value based on the binomial distribution.
(f) Do you reject or fail to reject the null hypothesis based on this p-value? The Supreme Court ruled that this disparity was small. Do you agree with this decision?

The following two exercises use Minitab. There is not a simple power function in R for one proportion. Minitab uses the normal approximation.

69. *Swain v. Alabama* (cont.)
Reconsider the *Swain v. Alabama* case where census data suggested that about 26% of those eligible for grand jury service were black. Suppose, due to some discriminating efforts, the rate at which blacks were selected for grand jury duty service was 0.25.
(a) Calculate the power of the test (with a sample size of $n = 1050$) against this alternative value ($\pi_a = 0.25$), using the $\alpha = 0.05$ level of significance. Include an interpretation of what the power calculation signifies.
(b) To verify your calculations using Minitab: Choose **Stat > Power and Sample Size > 1 Proportion**. Specify the sample size, the Comparison proportion (π_a), and the hypothesized proportion (π_0). Leave the Power values box empty. Under the "Options" button, specify the form of the alternative hypothesis, and keep 0.05 as the level of significance. Click the OK button twice. How does your calculation compare to Minitab's?
(c) Repeat (b) using 0.20 as the alternative value of the parameter. How does the power change for this new alternative value of the true underlying rate at which blacks are selected for grand jury duty? Provide an intuitive explanation of why this makes sense.

70. *Swain v. Alabama* **(cont.)**
Return to the context the previous exercise, where you used Minitab to perform the power calculation for two different alternative values of π, the underlying probability of a black person being selected for jury duty in Alabama in 1965.
(a) Choose **Stat > Power and Sample Size >1 Proportion** as before. In the Comparison proportion box, enter: .005:.995/.005. Minitab will create a graph of power as a function of the Comparison probabilities. Describe the behavior of this graph and explain why it makes intuitive sense. (You might want to "zoom in" on the more interesting part of the graph by double-clicking on the horizontal axis and then changing the minimum and maximum values for the scale.)
(b) Repeat (a) with a sample size of 550. Describe the behavior of the "power curve," explain why this behavior makes sense.
(c) Repeat (a) with a sample size of 150. Describe the behavior of the "power curve," and explain why this behavior makes sense.

71. Emotional Support?
In the mid-1980s sociologist Shere Hite undertook a study of women's attitudes toward relationships, love, and sex by distributing 100,000 questionnaires through women's groups. Of the 4500 women who returned the questionnaires, 96% said that they give more emotional support than they receive from their husbands or boyfriends. Around the same time, an ABC News/*Washington Post* poll surveyed a national random sample of 767 women, finding that 44% claimed to give more emotional support than they receive. Consider the population of interest for both surveys to be all American women.
(a) Identify (in words) the parameter of interest for both polls.
(b) With each poll, determine a 90%, 95%, and 99% confidence interval for the parameter. Calculate one of these confidence intervals by hand, but feel free to use technology (applet, R, Minitab) for the others.
(c) Which poll has the smaller margin-of-error? Explain why this poll has the smaller margin-of-error.
(d) Which poll's results do you think are more representative of the truth about the population of all American women? Explain.
(e) Which polling method do you think is more likely to be biased in a particular direction? Explain your answer, and also indicate whether you think that poll's statistic is an overestimate or underestimate of the population parameter.
(f) Determine the sample size that would be needed to estimate the population parameter to within ± 0.025 with 95% confidence. Use both 0.5 and the statistic from the ABC News/*Washington Post* poll to perform this calculation, and comment on how your answers differ.
(g) Based only on your confidence intervals from the ABC News/*Washington Post* poll, does 0.5 appear to be a plausible value for the proportion of all American women who would answer "yes" to this question? Explain.
(h) Conduct a (normal-based) significance test of whether 0.5 is a plausible value for the proportion of all American women who would answer "yes" to this question, based on the data from the ABC News/*Washington Post* poll. Report the hypotheses, test statistic, and p-value. State your test decision at the α = 0.10, 0.05, and 0.01 significance levels, and summarize your conclusion.

72. The *Literary Digest*
Recall from Investigation 1.13 that the *Literary Digest* conducted a large sample survey of 2.4 million people and found 57% indicating they would vote for Alf Landon in the upcoming 1936 election.
(a) Use technology to produce a 99.9% confidence interval for the proportion of all voters who were planning to vote for Alf Landon based on this sample data.
(b) Explain why this interval is so narrow.

(c) Explain why the results of this interval provided such an erroneous estimate (the actual value of π turned out to be 0.37 as Landon received only 37% of the vote in the nation election).

73. Hometown SUVs

Suppose that you want to estimate the proportion of vehicles on the road in your hometown that are sport utility vehicles (SUVs). You decide to stand at the intersection closest to your home between 7 and 8 A.M. every morning for a week, keeping track of how many vehicles go by and how many of them are SUVs.

(a) Identify the population, sample, parameter, and statistic (all in words) in this study.
(b) Give some reasons why your sampling method is probably not unbiased.

74. Hometown SUVs (cont.)

Reconsider the previous question. Now suppose that change your sampling plan and go to a local car dealership. You ask the manager to let you inspect a random sample of vehicles sold in the past year.

(a) Identify the sampling frame in this study.
(b) Is this sampling method likely to be unbiased for estimating the proportion of SUVs among all vehicles on the road in your town? Explain.

75. Representative Samples?

Suppose that you are asked to use the students in your current statistics class as a sample from the population of all students at your school. This is not literally a *random* sample, but whether or not this sample is *representative* of the population could depend on the variable of interest.
For each of the following variables, discuss whether you think the sample would be representative of the population. Briefly explain your answer.

(a) Gender
(b) Time spent sleeping last night
(c) Knowledge of statistics
(d) Political party affiliation
(e) Number of movies seen in past year
(f) Age

76. Representative Samples (cont.)

Reconsider the previous question. For each of the six variables listed there, classify it as categorical or quantitative.

77. Average Word Lengths

Suppose that you want to estimate the average length of a word (measured by number of letters) in this book.

(a) Is this number a parameter or a statistic? Explain. Also indicate what symbol would be commonly used to denote it.

Suppose that your sampling plan is to open the book haphazardly, set your finger on that page haphazardly, record the number of letters in the word that your finger lands on, and then repeat this process.

(b) Does this constitute a simple random sample? Explain.

(c) Is this sampling plan likely to be unbiased? If so, explain why. If not, indicate whether the sample mean is likely to over- or under-estimate the population mean, and explain your answer.

Now suppose that you decide to select a page at random, examine all of the words on that page, and calculate the average length of the words on that page.

(d) Is this number a parameter or a statistic? Explain. Also indicate what symbol would be commonly used to denote it.

(e) Answer (c) for this sampling plan.

(f) Explain how you could use a multi-stage sampling plan in this situation. Provide enough detail that someone else could implement your plan without further guidance from you.

78. Baseball Careers

Think about the average number of years in the career of a professional baseball player among all players who have played in the major leagues.

(a) Is this a parameter or a statistic? Explain, and indicate what symbol we would use to denote it.

(b) Suppose we estimate this average with the average number of years in the career of a player in the Baseball Hall of Fame. Is this likely to be an underestimate or an overestimate? Explain.

(c) If we estimate this average with the average years that current players have been playing, are we likely to obtain an overestimate or an underestimate? Explain which and why.

79. U.S. Senate

Suppose that we take the current members of the U.S. Senate as a sample from the population of all adult Americans. For each of the following, indicate whether you would expect the sample statistic to be an underestimate, and overestimate, or a fairly accurate estimate of the population parameter. Explain your answers.

(a) Proportion of women

(b) Average years of education

(c) Average annual income

(d) Standard deviation of ages

(e) Proportion that live below the poverty line

80. 2004 U.S. Open

Suppose that you want to gather data to test whether half of men's professional tennis matches last for the maximum possible number of sets. Before collecting the data, you do not have a conjecture for whether this proportion would be greater than or less than one-half if it does not equal one-half.

(a) State the appropriate null and alternative hypotheses, in symbols and in words.

In the 2004 U.S. Open, 15 of the 64 first round men's singles tennis matches lasted for 5 sets (the maximum number possible).

(b) Were these data gathered from a random sample of men's professional tennis matches in 2004? Were these data gathered from a random sample of matches in the 2004 U.S. Open? Explain.

(c) Do you have reason to suspect that this data collection method might be biased with regard to how long professional tennis matches last? Explain.

(d) Despite any concerns about the sampling method, use these data to conduct a test of the hypotheses in (a). Report the p-value, and describe how you calculated it. Do these data provide convincing evidence (at the $\alpha = 0.05$ level) against the null hypothesis?

81. 2004 U.S. Open (cont.)

Reconsider the previous exercise. Of the 64 women's singles tennis matches in the first round of the 2004 U.S. Open, 24 matches lasted for 3 sets (the maximum number possible). Suppose that you use these data to test whether half of all professional women's singles matches last for 3 sets.
(a) Without doing any calculations, explain whether the p-value of this test will be larger or smaller than the p-value based on the men's data, where 15 of 64 matches lasted for the maximum number of sets.
(b) Conduct the test based on the women's data. Report the p-value, and describe how you calculated it. Do these data provide convincing evidence (at the α = 0.05 level) against the null hypothesis?
(c) What is the smallest significance level at which the null hypothesis would be rejected? Explain. [*Hint*: Do not confine yourself only to the conventional significance levels.]
(d) Without calculating it, indicate whether you expect a 95% confidence interval for the underlying parameter to include the value .5. Explain.

82. Standard Deviation Properties

The standard deviation of a sample proportion is given by $\sqrt{\pi(1-\pi)/n}$.

(a) Explain what each of the three symbols ($\hat{p}$, π, and *n*) in this equation represent.
(b) Take the derivative of this function with respect to π and find the value of π that maximizes SD($\hat{p}$) for a fixed value of *n*. [*Hint*: You may also want to graph the function to verify you have found a maximum.]
(c) Now consider changing the sample size for a fixed value of π. Does the standard deviation decrease more by adding 500 subjects to a sample size of 500 or to sample size of 2500? Explain.
(d) Take the (first and) second derivative of SD($\hat{p}$) as a function of *n* to determine whether this function of *n* is concave up or concave down.
(e) Explain what your analysis in (b) and (c) reveal about the "diminishing returns" of increasing sample size. [*Hint*: you may want to create and examine at a graph of SD($\hat{p}$) vs. *n* for values of *n* from say 0 to 3000.]

83. Margin-of-Error Properties

The margin-of-error of a confidence interval for a population proportion π using the Wald procedure is $z*\sqrt{\pi(1-\pi)/n}$. Some books recommend a shortcut formula that approximates this margin-of-error for a 95% CI for π quite simply by $1/\sqrt{n}$.

(a) Explain why this is a reasonable approximation. [*Hint*: What simplification can you make in the given formula when you are using a 95% confidence level? What about different values of π?]
(b) Show that this approximation is conservative, in that it slightly overestimates the actual margin-of-error.
(c) Reconsider Practice Problem 1.10 and re-answer part (b) – solving for the necessary sample size using this approximation.
(d) Suggest two different ways (that the researcher has direct control over) to reduce the margin-of-error in a study.

84. Margin-of-Error Properties (cont.)

Consider the margin-of-error of a confidence interval for a population proportion π using the Wald

procedure: $z^*\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$. Suppose that you want the margin-of-error to be no larger than some pre-

specified error bound, *M*. That is, we want to satisfy the inequality $z^*\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \leq M$.

(a) Rearrange the terms in the inequality to solve for the sample size *n* necessary to achieve such a margin-of-error, as a function of z^*, *M*, and $\hat{p}$.
(b) Is the expression in (a) an increasing function of z^*, a decreasing function of z^*, or neither? Consequently, is it an increasing function of the confidence level, a decreasing function of the confidence level, or neither? Explain why this makes intuitive sense.
(c) Is the expression in (a) an increasing function of *M*, a decreasing function of *M*, or neither? Explain why this makes intuitive sense.
(d) Is the expression in (a) an increasing function of $\hat{p}$, a decreasing function of $\hat{p}$, or neither? Use calculus to determine the value of $\hat{p}$ that maximizes this function.
(e) Explain why your answer to (d) suggests that the conservative approach to sample size determination is to use 0.5 as a guess for $\hat{p}$.

85. Margin-of-Error and Sample Size
Consider again the margin-of-error of a confidence interval for a population proportion π using the Wald

procedure: $z^*\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$.

(a) Determine whether this is an increasing or decreasing function of *n*, or neither.
(b) Take the second derivative of this function of *n* to determine whether this function of *n* is concave up or concave down.
(c) Sketch the behavior of this margin-of-error as a function of *n* for 95% confidence with $\hat{p}$ = 0.5.
(d) Does the margin-of-error decrease more by adding 500 subjects to a sample size of 500 or to a sample size of 2500? Explain.
(e) Explain what your analysis reveals about the "diminishing returns" of increasing sample size.

86. Reducing the Margin-of-Error
Suggest two different ways for reducing the *margin-of-error* in a confidence interval for a population proportion π.

87. Sample Size Determination
Suppose you want to estimate the proportion of students at a very large university who are nearsighted. The prevalence of nearsightedness in the general U.S. population is 45%. Using this as a preliminary guess of π, how many students would need to be included in a random sample if you wanted the margin-of-error of a 95% confidence interval to be at most 1%?

88. The Wilson Adjustment

There are two probability rules concerning a constant times a random variable: $E(cX) = \underline{c}E(X)$ and $SD(cX) = c \cdot SD(X)$. Similar rules for the sum of a random variable X and a constant c are: $E(X + c) = E(X) + c$ and $SD(X + c) = SD(X)$.

(a) Explain why each of these last two rules makes intuitive sense.

(b) Let the random variable X have a binomial distribution with parameters n and π, let the random variable $\hat{p} = \dfrac{X}{n}$ represent the sample proportion, and let the random variable $\tilde{p}$ represent the

Wilson adjustment: $\tilde{p} = \dfrac{X + 2}{n + 4}$. Rewrite $\tilde{p}$ in the form $cX + k$, where c and k are constants.

(c) Use your expression in (b) and both sets of rules to determine the expected value of $\tilde{p}$. Is this expected value equal to π?

(d) Use your expression in (b) and both sets of rules to determine the standard deviation of $\tilde{p}$.

(e) How does the standard deviation of $\tilde{p}$ compare to the standard deviation of $\hat{p}$?

89. Technical Conditions

The condition that you have studied for the reasonableness of the normal approximation to the binomial distribution is: $n\pi \geq 10$ and $n(1 - \pi) \geq 10$. An alternative that has been proposed is:
$$n\pi(1-\pi) \geq 5.$$

(a) Suppose that $\pi = 0.5$. According to the conventional condition, how large would the sample size n have to be in order for the normal approximation to be reasonable?

(b) Still in the $\pi = 0.5$ case, how large does this alternative condition say that the sample size n must be in order for the normal approximation to be reasonable? How does this compare to your answer from (a)?

(c) Repeat (a) and (b) for the $\pi = 0.25$ case.

(d) Repeat (a) and (b) for the $\pi = 0.9$ case.

(e) For the $n\pi (1 - \pi) \geq 5$ condition, express the required sample size n as a function of the success probability π. Graph this function, and comment on its behavior. [*Hint*: Feel free to use technology, and you might use values of π from 0 to 1 in multiples of 0.01.]

(f) Repeat (e) for the $n\pi \geq 10$ and $n(1 - \pi) \geq 10$ condition. [*Hint*: Remember that n must satisfy both conditions.]

(g) Write a few sentences comparing these conditions. Which seems to require a larger sample size? Does this vary depending on the value of π?

90. Sample Proportions

Let the random variables X and Y have binomial distributions with sample sizes n_1 and n_2, respectively, and let $\hat{p}_1$ and $\hat{p}_2$ represent the corresponding sample proportions of success. Recall that the pooled estimate of the underlying probability is $\hat{p} = \dfrac{X + Y}{n_1 + n_2}$.

(a) Show that $\hat{p}$ can be re-written as $\dfrac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$.

(b) Rewrite $\hat{p}$ as a weighted average of $\hat{p}_1$ and $\hat{p}_2$. In other words, express $\hat{p}$ as $\alpha \hat{p}_1 + (1 - \alpha)\hat{p}_2$ for an appropriate value of α.

(c) Describe the two circumstances under which $\hat{p}$ would equal the (ordinary) average of $\hat{p}_1$ and $\hat{p}_2$.

91. Inference Subtleties
The following questions address some finer distinctions about the inference procedures you learned in this chapter.
(a) Does the Central Limit Theorem indicate that all samples follow a normal distribution if the sample size is large enough? Explain.
(b) Suppose that the observational units in a study are people in your home state, and the variable of interest in a study is number of siblings. If the sample size is chosen to be in the thousands, would a histogram of the sample data follow a normal distribution? Explain.
(c) If you reject a hypothesis at the 0.05 level, what can you say about whether it would be rejected at the 0.07 level? How about at the0 .03 level? Explain.
(d) If a 95% confidence interval for a population parameter contains a particular value, what can you say about whether a 93% confidence interval would contain that value? How about a 97% confidence interval? Explain.
(e) If a 95% confidence interval for a population parameter contains a particular value, is it always true that you will fail to reject this hypothesized value in favor of a one-sided alternative? Explain.

92. Current Smokers
The 2002 National Health Interview Survey (NHIS) took a representative sample of 31,044 American adults using a multi-stage cluster design. One of their findings was that 22.5% of the individuals (age 18 and over) sampled identified themselves as current smokers. The report listed the standard error of this statistic as 0.0032.
(a) If the sampling design had been a simple random sample, what would the standard error have been?
(b) Is the reported standard error from the multistage cluster design larger or smaller than the standard error from a simple random sample of the same size? Explain why makes sense.
(c) Identify in words the parameter of interest in this study.
(d) Use the reported standard error to produce a 99% confidence interval for this parameter.
(e) Would a test of whether the data provide evidence that less than 25% of American adults were smokers in 2002 produce a significant result at the $\alpha = 0.01$ level? Explain. [*Hint*: Remember to use the reported standard error.]

93. Current Smokers (cont.)
Reconsider the smoking data from the 2002 NHIS from the previous question. The report also mentioned that 25.2% of the males interviewed and 20.0% of the females interviewed identified themselves as current smokers.
(a) Do you expect the standard errors of these statistics to be less than, greater than, or equal to the 0.0032 standard error of the previous question? Explain.
The report listed the standard errors to be 0.0047 for the males and 0.0039 for the females.
(b) Do you need to reconsider your answer to (a) in light of this information?
(c) Answer questions (c)–(e) of the previous question separately for males and for females.

94. Heart Transplant Mortality (cont.)
Reconsider the situation of Investigation 1.11, in which you tested whether the true mortality rate of heart transplant procedures at the hospital exceeded 0.15. Recall that data on the previous 361 patients revealed 71 deaths.
(a) Do these data provide very strong evidence that the underlying mortality rate exceeded 0.15? Would you reject the null hypothesis at the $\alpha = 0.01$ significance level? Explain, based on the p-value of the test.
(b) Repeat (a), if there had been 70 deaths, rather than 71, in the sample of 361 patients.

(c) Repeat (a), if there had been 60 deaths, rather than 71, in the sample of 361 patients.
(d) In which two of these three cases do you make the same decision at the $\alpha = 0.01$ significance level?
(e) In which two of these three cases are the sample results most similar?
(f) Explain why your answers to (d) and (e) have convinced many statisticians to find the p-value more informative than only a statement of whether a null hypothesis is rejected at one particular significance level.

95. High Anxiety
The Anxiety Disorders Association of America claims on its website
(http://www.adaa.org/mediaroom/index.cfm, December 4, 2004) that 13.3% of adults suffer from some form of anxiety disorder.
(a) Describe how you might design a study to assess this claim.
(b) State the hypotheses that you would test, both in symbols and in words.
(c) If you want to estimate the population proportion who suffer from an anxiety disorder to within ± 0.05 with 95% confidence, how large a sample would be necessary? (Use the association's claim as a reasonable initial estimate of this proportion.)
(d) How would your answer to (c) change (if it all) if you wanted to estimate the proportion of women who suffer from an anxiety disorder? In other words, would the sample size need to be larger or smaller or the same for this population?
(e) How would your answer to (c) change (if it all) if you wanted to estimate the population proportion who suffer from an anxiety disorder in your state?
(f) Re-answer (c) using the more conservative estimate that the proportion is 0.5 rather than the association's claim of 0.133. Is the required sample size larger now? Does the increase in sample size appear to be substantial?

96. Leaving Office?
The news website MSNBC.com reports that one week during the Clinton-Lewinsky scandal, readers of the site were invited to vote in an unscientific poll that asked whether President Clinton should leave office. The site received over 200,000 votes, of which 73% said "yes."
(a) Describe the population and parameter of interest here.
(b) Use these sample data to determine a 99% confidence interval for the proportion of adult Americans who felt during the week in question that Clinton should leave office. Also report the margin-of-error.
(c) Do you think that this interval does a reasonable job of estimating the population proportion who felt that Clinton should leave office? Explain.
During the same week, an NBC News–Wall Street Journal poll contacted a random sample of 2005 people, with 34% answering that Clinton should leave office.
(d) Re-answer questions (b)–(c) based on these sample data.
(e) Which of these two intervals (the one based on the MSNBC.com poll or the one based on the NBC News- Wall Street Journal poll) is narrower? Explain why that makes sense.
(f) Do these two intervals overlap? Are they similar at all?
(g) Which interval do you think provides a more reasonable estimate of the proportion of adult Americans who felt that Clinton should leave office? Explain.

97. Prescribing Placebos
Many medical studies compare a new treatment against a placebo, because many patients respond well even to a placebo. This has led some to question whether there might be circumstances in which it would be appropriate for a physician to administer a placebo to a patient. Nitzan and Lichtenberg (2004)

surveyed 89 physicians and head nurses in Israel, chosen from across all medical and surgery units at two hospitals. They found that 53 admitted administering placebos to patients.
(a) Identify the population of interest and the sample selected.
(b) Identify the parameter and statistic (both in words and with a symbol). Also calculate the value of the statistic.
(c) How large must the population (total number of physicians and head nurses in Israel) be in order for the binomial model to be a reasonable one here? Explain.
(d) Use technology to determine a 95% binomial confidence interval for π, the proportion of all physicians and head nurses in Israel who prescribe placebos to patients.
(e) Now determine a 90% binomial confidence interval for π, and comment on how it compares to the 95% interval.
(f) Would you use these intervals to estimate the proportion of American physicians who prescribe placebos? Explain.

98. Prescribing Placebos (cont.)
Reconsider the previous exercise about prescribing placebos. The researchers stated that they had expected no more than 10% of clinicians to use placebos.
(a) State the null and alternative hypotheses, in symbols and in words, for testing whether the sample data provide evidence that more than 10% of clinicians have used placebos (i.e., to invalidate what the researchers expected).
(b) Conduct a binomial test of the hypotheses that you stated in (a). Report the p-value and summarize your conclusion.
(c) Based on the confidence intervals that you determined in the previous exercise, what can you say about the p-value for conducting a two-sided test of whether the population proportion differs significantly from 0.5? Explain. (Do not conduct the test; base your answer on the confidence intervals.)

99. *Cohen v. Brown University*
In 1991, a suit was filed against Brown University after Brown terminated funding for its women's gymnastics and volleyball teams and its men's water polo and golf teams. The suit charged that Brown was violating Title IX of the Education Amendments of 1972, the federal law that prohibits sex discrimination by all educational institutions receiving federal funds. This requires men and women to have equivalent opportunities for participation. A main component of the plaintiff's case was that while 51% of the undergraduate student body was women, only 38% of the 897 students engaged in intercollegiate athletics were women. If there is no gender discrepancy, then Title IX assumes that the proportion of women athletes should be similar to the proportion of women in the overall student body. However, we know the sample result can deviate from this population proportion "just by chance." Suppose we were randomly selecting students to be athletes, the question is whether this random process could lead to such a disparity in these proportions. Although we know the proportion of women in the population of all university students to be 0.51, we don't know the probability of an athlete being female. Let π refer to the probability of a Brown university athlete being female.
(a) What are the observational units and variable of interest in this study? Also specify the population or process of interest. [Note: Although we have not actually taken a random sample here, we will assume the sample of 897 students is representative of this overall process.]
(b) State a null and an alternative hypothesis about the value of π (in symbols and in words).
(c) We can assess the strength of evidence against the null hypothesis by treating the 897 current athletes as a random sample from the process of athlete determination at Brown. Carry out a one-proportion z-test to assess whether the observed sample proportion ($\hat{p} = 0.38$) is surprising under the null

hypothesis. Check and comment on the technical conditions of this procedure. Would it be beneficial to include a continuity correction? Explain.

(d) Based on this p-value, what conclusion do you draw about whether this discrepancy could have arisen by chance? In other words, does your analysis suggest that the proportion of women involved in intercollegiate athletics is significantly lower than the proportion of women students at the university? Explain.

Additional Practice and Applications with the Normal Distribution

100. Normal Probability Density Functions
Consider the standard normal probability density function, and let Z denote a standard normal random variable. [*Hint*: Remember that "standard" indicates that $\mu = 0$ and $\sigma = 1$, so the function

is $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$.]

(a) Evaluate this function at $z = 0$.
(b) Does your answer to (a) represent the probability that a standard normal random variable equals zero, $P(Z = 0)$? [*Hint*: You might want to draw a sketch, and remember what probabilities correspond to with a density curve.]
(c) Now consider a normal model for a random variable X with $\mu = 0$ and $\sigma = 5$. Write out the probability density function in this case, and then evaluate it at $x = 0$. Does this represent the probability that X = 0 with this model? Explain.
(d) Now consider a normal model with $\mu = 0$ and $\sigma = 0.1$. Evaluate the probability density function at $x = 0$. Does this represent the probability that X = 0 with this model? Explain what's different about this answer and the previous ones.
(e) Explain how it is possible that evaluating a legitimate pdf at a particular value of x can yield a value greater than one, yet probabilities calculated from legitimate pdfs are never greater than one.

101. Normal Probability Density Curves
Consider the standard normal probability density function.
(a) Use calculus to determine where the maximum value occurs with the standard normal probability density function. [*Hint*: Take the derivative with respect to x, set the derivative equal to zero, and solve for x. Then check the second derivative to make sure that you have found the value of x where the *maximum* occurs.]
(b) Use calculus to determine where the inflection points occur with the standard normal probability density function.
(c) Repeat (a) and (b) for the normal pdf with parameters μ and σ. Show that the maximum occurs at $x = \mu$ and the inflection points occur at $x = \mu - \sigma$ and x = $\mu + \sigma$.

102. Modeling Pregnancy Durations
According to the National Vital Statistics Reports, there were 4,130,665 live births in the United States in 2009. The report lists 30,567 pre-term deliveries, meaning the pregnancy lasted for less than 37 weeks, whereas 228,839 lasted for more than 42 weeks ("post-term deliveries"). If we want to model pregnancy durations with a normal distribution, we can use this information to determine the values of the parameters μ and σ.
(a) Of the pregnancies with known gestation periods, what proportion resulted in pre-term deliveries? What proportion resulted in post-term deliveries?
(b) Draw a well-labeled sketch of a normal curve to model these pregnancy durations, with parameters μ and σ still to be determined, but with areas corresponding roughly to the proportions calculated in (a).
(c) Determine the z-scores corresponding to the values 37 weeks and 42 weeks, in order for the proportions calculated in (a) to hold.
(d) Set $(37-\mu)/\sigma$ and $(42-\mu)/\sigma$ equal to these z-scores. Then solve this system of two equations in two unknowns for μ and σ.

103. Candy Bar Weights
Suppose that a candy bar wrapper reports the weight of the candy bar to be 1.55 ounces. Suppose that the actual weights of the candy bars vary according to a normal distribution with mean $\mu = 1.60$ ounces and standard deviation $\sigma = 0.02$ ounces.
(a) Draw a well-labeled sketch of this model for the distribution of candy bar weights.
(b) According to the model, what proportion of candy bars will weigh less than the wrapper advertises? Now suppose that the manufacturer wants only 0.1% of the candy bars to weigh less than what the wrapper advertises. At least one of three things must change: the weight listed on the wrapper, the mean weight of the bars in the production process, or the standard deviation of the weights of the bars in the production process.
(c) To accomplish the manufacturer's goal, what weight should be listed on the wrapper, assuming that the mean and standard deviation of the weights in the production process do not change?
(d) What should the mean weight in the production process be changed to, if the weight listed on the wrapper is to remain 1.55 ounces and the standard deviation of the bar weights is not to change?
(e) What should the standard deviation of the candy bar weights in the production process be changed to, if the weight listed on the wrapper is to remain 1.55 ounces and the mean of the bar weights is not to change?
(f) Which of these three options (changing the label value, the mean, or the standard deviation) do you suspect is/are under the manufacturer's control? Explain.
(g) If the manufacturer wants only 0.01% to weigh less than advertised, in what direction would the mean and/or standard deviation need to change? Give an intuitive explanation.

104. Candy Bar Weights (cont.)
Reconsider the previous question, with the original specifications that the wrapper lists the weight as 1.55 ounces and the actual weights of the candy bars vary according to a normal distribution with mean $\mu = 1.60$ ounces and standard deviation $\sigma = 0.02$ ounces.
(a) In a random sample of 10 candy bars, what is the probability that at least one weighs less than the advertised weight? [*Hint*: Consider the random variable Y = number of the ten bars that weigh less than advertised. What probability distribution does Y have?]
(b) If a random sample of 10 candy bars revealed that 3 weighed less than advertised, would you have reason to doubt that the production process is operating according to its specifications? Explain. [*Hint*: What is the probability of a result at least this extreme occurring by chance alone? Would you consider this result surprising?]

105. Paint Drying Time
Suppose that the drying time for a certain type of paint under specified test conditions is known to be normally distributed with mean 75 minutes and standard deviation 5 minutes. Suppose that chemists have devised a new additive that they hope will reduce the mean drying time (without changing the standard deviation). Suppose that a test is conducted to measure the drying time for a test specimen, and suppose that company executives decide that they will be convinced that the additive is effective only if the drying time on this specimen is less than 70 minutes.
(a) If the additive actually has no effect at all on the drying time, what is the probability that the company executives will mistakenly conclude that it is effective? Include a well-labeled sketch with your calculation.
Now suppose that the additive really is effective and that it reduces the mean drying time to 65 minutes, without changing the standard deviation of 5 minutes.

(b) Draw a well-labeled sketch of the two normal curves on the same scale. (You can sketch these by hand, or you can copy from technology. To get both curves to appear in the applet, check the box for the second mean and sd row and enter the second set of values.)

(c) What is the probability that this test will fail to convince the executives that the additive is effective, even though it actually is?

(d) If you want alter the cut-off value from 70 in order to reduce the error probability in (a) to 0.05, what cut-off value should you choose?

(e) Using this new cut-off value found in (d), what is the probability that the test will fail to convince the executives that the additive is effective, even though it actually is?

(f) How does the probability in (e) compare to that in (c)? Explain why this makes sense.

(g) Suppose that the additive not only reduced the mean drying time to 65 minutes but also reduced the standard deviation to 2 minutes. Re-calculate the error probability in (e). Comment on how it has changed, and explain why this makes sense.

106. Normal Paternity

An expert witness in a paternity suit testifies that the length (in days) of pregnancy (the time from conception to delivery of the child) is approximately normally distributed with mean $\mu = 270$ days and standard deviation $\sigma = 10$ days. The defendant in the suit is able to prove that he was out of the country during a period that began 280 days before the birth of the child and ended 230 days before the birth of the child.

(a) If the defendant was the father of the child, what is the probability that the mother could have had the very long or the very short pregnancy indicated by the testimony? (Make sure you include a well-labeled sketch of the distribution and the areas of interest. Include any technology output or screen captures.)

(b) If you were the judge in this case, based on the calculation in (a) alone, how would you rule? Explain.

107. Modeling the Body Mass Index

Suppose that the body mass index (BMI) of healthy American males follows a normal distribution with mean 24.5 and standard deviation 3.0 and that the BMI of healthy American females follows a normal distribution with mean 22.5 and standard deviation 3.0.

(a) Sketch (and label) these normal curves on the same scale.

(b) What proportion of healthy American males have a BMI above 25? How about females?

(c) What proportion of healthy American males have a BMI below 20? How about females?

(d) If you learn that an individual has a BMI of 19.6, would you suspect that the person is male or female? Explain.

108. Filling Cereal Boxes

Suppose that a cereal manufacturer advertises that its cereal boxes contain 16 ounces of cereal. The actual weight of the cereal put into boxes by machines follows a normal distribution with mean 16.10 ounces and standard deviation 0.08 ounces.

(a) Produce a well-labeled sketch of this distribution. (Feel free to use technology.)

(b) How many standard deviations below the mean is the advertised weight? (Show how you calculate this.)

(c) What proportion of cereal boxes will be filled with less than the advertised weight? (Also indicate the area corresponding to this proportion on your sketch.)

(d) Determine the weight such that only 1% of cereal boxes weigh less than that weight.

Now suppose that the company executives determine that your answer to (c) is an unacceptably large

proportion of boxes that weigh less than advertised. They want to adjust the process of putting cereal into boxes so that only 1% of cereal boxes weigh less than the advertised weight.

(e) Determine the *z*-score for which only 1% of the values in a normal distribution fall below that *z*-score. Suppose for now that the standard deviation of the box-filling weights is not to be changed from 0.08 ounces.

(f) What value of the mean weight should be used in order to achieve their goal that only 1% of cereal boxes weigh less than the advertised weight? (Show/explain your work in this calculation. Make use of your answer to part e.)

(g) How does this adjusted mean weight compare to the original mean? Explain why the company executives might be displeased about adjusting the mean weight of the box-filling process in this way.

Now suppose that the company executives are unwilling to increase the mean weight of the box-filling process from 16.10 ounces.

(h) What value of the standard deviation (SD) should be used in order to achieve their goal that only 1% of cereal boxes weigh less than the advertised weight? (Show/explain your work in this calculation. Make use of your answer to part e.)

(i) How does this adjusted SD compare to the original SD? By what percentage does the SD need to be decreased in order to achieve the goal?

Now suppose that the company executives want to have only 0.5% of cereal boxes weigh less than advertised, and they also want only 0.5% of cereal boxes to weigh more than 16.2 ounces.

(j) Determine the values of the mean and SD that achieve this pair of goals. Show/explain your work in this calculation.

(k) Will this pair of goals require an even more precise (less variable) box-filling process than the previous goal? Explain.