Practice Problems

3.3.3 Halloween Treat Choices (cont.)

Reconsider Investigation 3.3.4, and continue to let π represent the probability that a child would choose the toy.

(a) Suppose we had used the following hypotheses: H_0: $\pi = .5$ vs. H_a: $\pi < .5$. What is the *p*-value for this set of hypotheses? How does this *p*-value compare to the two-sided *p*-value from Investigation 3.3.4.

(b) Suppose we had used the following hypotheses: H_0: $\pi = .5$ vs. H_a: $\pi > .5$. What is this *p*-value for this set of hypotheses? How does this *p*-value compare to the one-sided *p*-value from part (a)?

3.3.4 Armrest Battles

In a study reported in 1982 by Hai, Khairullah and Coulmas, researchers observed 426 pairs of passengers in "mixed-sex" seating arrangements on airplanes to see if either person used the joint armrest. Observations were made after a beverage or a meal was served. Passengers who were asleep or obviously couples were not counted. Data were collected by noting whether the man, the woman, both, or neither was using the armrest. Of the 426 pairs, the man used the armrest 284 times, the woman 57 times, both 37 times, and neither 48 times. We will consider only the 341 pairs for which one person or the other used the armrest.

(a) State the hypotheses for testing whether men are more likely than women to use the shared armrest. (Be sure to interpret what π represents in this context.)

(b) Use the binomial distribution to calculate the *p*-value of this test.

(c) Identify a potential confounding variable to explain the higher armrest use by males in this study.

3.3.5 Kissing the Right Way (cont.)

Reconsider Investigation 3.3.5 and continue to let π represent the probability that a kissing couple turns to the right.

(a) Use the applet to determine the two-sided *p*-value against the alternative hypothesis that π differs from .5. (*Hint*: Use ≥ 80.)

(b) Is your *p*-value in (a) convincing evidence that π differs from .5?

(c) Repeat (a) and (b) for $\pi = .75$. (Use ≤ 80.)

(d) Repeat (a) and (b) for $\pi = .70$.

INVESTIGATION 3.3.6 KISSING THE RIGHT WAY (CONT.)

In the previous investigation, you learned how to decide whether a hypothesized value of the population parameter was plausible based on a one-sided or a two-sided *p*-value. The two-sided *p*-value was used when you did not have a prior suspicion or interest in whether the hypothesized value was too large or too small. In fact, in many studies we may not even really have a hypothesized value, but are more interested in using the sample data to estimate the value of the population parameter.

a. If you observed $\hat{p} = 80/124$ couples turn to the right, what is your best guess for the value of π?

b. Do you believe π is exactly equal to the value specified in (a)? Do you think it is close? Explain.

We can employ a "trial-and-error" type of approach to determine which values of π appear plausible based on what we observed in the sample. This involves testing different values of π and seeing whether the corresponding two-sided p-value is larger than .05. That is, we will consider a value plausible for π if it does not make our sample result look surprising.

c. You found in the previous investigation and practice problem that .5 and .75 do not appear to be plausible values for π, but .67 and .70 do because the two-sided p-values are larger than .05. Determine the values of π such that observing 80 of 124 successes or a result more extreme occurs in at least 5% of samples. (*Hints:* Use values of π that are multiples of .01 until you can find the boundaries where the two-sided p-values change from below .05 to above .05.)

DEFINITION A *confidence interval* specifies the plausible values of the parameter based on the sample result.

Confidence intervals are an additional or alternative step to a significance test, which tells us whether or not we have strong evidence against one particular value for the parameter. Again, statistical software packages employ slightly different algorithms for finding the binomial confidence interval. This investigation should help you understand how to interpret the resulting confidence interval given by the software. What you found in (c) will be called a 95% *confidence level* since it was derived using the 5% level of significance.

d. If you were to repeat (c) but using .01 rather than .05 as the criterion for plausibility, would this "99% confidence interval" include more or fewer values than the one based on the .05 criterion? Explain your reasoning.

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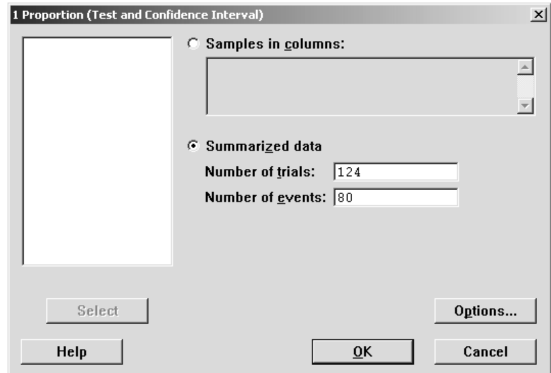
Minitab Detour

You can use Minitab to perform these p-value and confidence interval calculations for you. Choose Stat > Basic Statistics > 1 Proportion, and choose the "Summarized data" option. Specify n as the "number of trials" (124 here) and the observed number of successes as the "number of events" (80 here). Then if you click the "Options" button, you can change the "confidence level", the "test proportion," and the direction. To obtain a confidence interval, the direction needs to be set to "not equal to."



e. Use Minitab to verify the confidence interval endpoints that you found in (d).

f. Use Minitab to verify the exact two-sided p-value that you found for testing $\pi = .667$ against the alternative hypothesis that $\pi \neq .667$ based on observing 80 right-turning couples out of 124.

g. Use Minitab to verify the exact one-sided p-value that you found for testing $\pi = .5$ against the alternative hypothesis that $\pi > .5$.

Study Conclusions

The researchers are assuming they have a representative sample from a process and want to determine π, the probability that a kissing couple turns to the right. Based on a random sample of 124 observations, we estimate π to be close to $80/124 = .645$. However, we know there is some sampling variability, so we want to find an interval of values that appear to be plausible values of π. We do this by finding the values of π for which the two-sided *p*-value is greater than .05. These are all the values of the population parameter such that our sample result is not overly surprising. You should have found this "95% confidence" interval to be approximately .557 to .726 (results from using Minitab or the applet will differ slightly). Thus, based on these sample results, we are "confident" that the actual value of π is between .56 and .73, that is, that between 56% and 73% of all kissing couples turn to the right. A 99% confidence interval for π extends from .53 and .75 and therefore includes more values than a 95% interval. The higher level of confidence requires more "room for error." You will learn more about confidence intervals in the next chapter.

Discussion In this activity you have learned a second type of "statistical inference"—based on the sample statistic—providing a range of plausible values for the population parameter. Confidence intervals provide a nice companion to tests of significance and are also very useful by themselves. Although a test of significance allows to you test a specific hypothesized value, if you reject the null hypothesis, the test of significance provides no information as to *how different* the actual parameter is from the hypothesized value. The confidence interval provides an estimate (with bounds) of the actual value of the parameter.

In fact, there is a type of *duality* between confidence intervals and tests of significance. The confidence interval is the set of values for which we would fail to reject the null hypothesis in favor of the *two-sided* alternative. In fact, Minitab's algorithm for the two-sided *p*-value is obtained by maintaining this correspondence (the small *p*-value's approach may lead to small departures from this duality). So we can interpret the confidence interval as the set of plausible values for the parameter in that they are the values such that our observed sample result would not be surprising.

In summarizing your results, remember to always conclude with an answer to the research question in context. The decision to reject or fail to reject H_0 should never be your last statement. Similarly, the determination of a confidence interval should never be the last statement. Include at least one more sentence interpreting the results in the context of the problem, for example, what is supposed to be in the confidence interval and how confident are you that it is!

SUMMARY OF EXACT BINOMIAL INFERENCE

Let X represent the number of successes in the sample and π the probability of success for the process. If X satisfies the criterion for a binomial random variable (two outcomes, fixed probability of success, independent trials),

To test H_0: $\pi = \pi_0$

We can calculate a *p*-value based on the binomial distribution with parameters n and π_0. The *p*-value can be one-sided or two-sided based on the statement of the research conjecture:

$$\text{if } H_a\text{: } \pi > \pi_0\text{: } p\text{-value} = P(X \geq \textit{observed})$$

$$\text{if } H_a\text{: } \pi < \pi_0\text{: } p\text{-value} = P(X \leq \textit{observed})$$

if H_a: $\pi \neq \pi_0$: *p*-value = sum of both tail probabilities using method like "small *p*-values"

C% confidence interval for π

The set of values such that the two-sided *p*-value based on the observed count is larger than the $(1 - C)/2$ level of significance.