Consider our Netherlands data (Day 9)

Null Model:

ML	**REML**
Random effects:	Random effects:
Groups Name Variance Std.Dev.	Groups Name Variance Std.Dev.
schoolnr (Intercept) 18.13 4.257	schoolnr (Intercept) 18.24 4.271
Residual 62.85 7.928	Residual 62.85 7.928
Number of obs: 3758, groups: schoolnr, 211	Number of obs: 3758, groups: schoolnr, 211
Fixed effects:	Fixed effects:
Estimate Std. Error t value	Estimate Std. Error t value
(Intercept) 41.0046 0.3249 126.2	(Intercept) 41.0038 0.3257 125.9

The model fit using Maximum Likelihood estimates the "total variation" to be 18.13 + 62.85 = 80.98. (ICC = 18.13/80.98 = .224).

The model fit using REML estimates the "total variation" to be 18.24 + 62.85 = 81.09. (ICC = 18.23/81.09 = .225).

Added verbal IQ to the model

ML	**REML**
Random effects:	Random effects:
Groups Name Variance Std.Dev.	Groups Name Variance Std.Dev.
schoolnr (Intercept) 9.845 3.138	schoolnr (Intercept) 9.909 3.148
Residual 40.469 6.362	Residual 40.479 6.362
Number of obs: 3758, groups: schoolnr, 211	Number of obs: 3758, groups: schoolnr, 211
Fixed effects:	Fixed effects:
Estimate Std. Error t value	Estimate Std. Error t value
(Intercept) 41.05488 0.24339 168.68	(Intercept) 41.05442 0.24402 168.24
IQ_verb 2.50744 0.05438 46.11	IQ_verb 2.50722 0.05439 46.09

Using the REML values

When we added verbal IQ to the model, we explained

(62.85 – 40.479)/62.85 => 35.59% of the Level 1 variation

(18.13 – 9.845)/18.13 => 45.67% of Level 2 variation (school to school)

```
> performance::r2(model1, by_group=TRUE)
# Explained Variance by Level

Level    |    R2
----------------
Level 1  | 0.356
schoolnr | 0.457
```

These are the calculations I like to focus on. But you can get negative values!

Thinking in terms of the total variation:

> (81.09 – (9.909 + 40.479))/81.09 = 0.3786, 37.86% of the total variation in the language scores.
>
> The new ICC is (9.909/(9.909 + 40.479) = 0.197 (slight decrease)

Performance Package

To understand some of the numbers output by the Performance package, we need to understand the "variance of the fixed effects" = $var(X\hat{\beta}) = \Sigma(\hat{y}_i - \bar{\hat{y}})^2/(df)$. This measures how much the fitted values, based only on the fixed effects not the random effects, vary across this dataset/predictor variable values. Notice how $\hat{\tau}^2 + \hat{\sigma}^2$ is much smaller in Model 1 than the total variance in Y. They represent the variation explained by the groups or the unexplained variation. The difference is akin to the variation explained by the fixed effects.

> var(model.matrix(model1) %*% fixef(model1)) [,1] [1,] 26.18429	sum((model.matrix(model1)%*% fixef(model1) - 41.1652)^2) [1] 98356.62 > 98374/3757 26.18 where 41.1652 = mean(model.matrix(model1) %*% fixef(model1))

So then we could say the total variation is = $\sigma^2 + \tau^2 + var(XB)$. So for model 1, we find Total variance = 40.479 + 9.909 + 26.18 = 76.57

• "Marginal R^2" measures the variance explained by the new variable as a proportion of the sum of all three of these variances.	26.179/76.57 = .342 "the fixed effects explain …"	> performance::r2(model1) # R2 for Mixed Models Conditional R2: 0.471 Marginal R2: 0.342
• "Conditional R^2" measures the proportion of the total variance explained by both the fixed and random effects in the model.	(26.18 + 9.909)/76.57 = 0.471 "the fixed and random effects explain …"	
• The adjusted ICC is what we would calculate "by hand" which just uses the variance components after adding the covariate into the model.	9.909/(9.909 + 40.479) = 0.197	performance::icc(model1) # Intraclass Correlation Coefficient Adjusted ICC: 0.197 Unadjusted ICC: 0.129
• The unadjusted ICC is the difference between the conditional and marginal R^2values (the contribution of the random effects)	(47.1-34.2) = 9.909/76.57 = 0.129	

- We will focus more on the adjusted ICC, if that. Of real interest to us is the ICC from the null model, but you can look at the ICC in other models to see how that has impacted the "unexplained" group to group variation.

    - The adjusted intraclass correlation coefficient is often smaller than the "raw" (null model) intraclass correlation coefficient (observations are less correlated when you have accounted for some of their similarities?).
- We will focus on the variance explained at each level (comparing the reduction in τ^2 or σ^2 (ignoring the variance of the fixed effects) (see by_group = TRUE) or $\tau^2 + \sigma^2$

Once we have added a fixed effect to the model, one way to get all three variances out of R is using the 'insight' package:

```
> insight::get_variance(model1REML)
$var.fixed
[1] 26.17956
$var.random
[1] 9.908699
$var.residual
[1] 40.4794
```