

Stat 414 – Day 11 Output

Example 1: FEV

Model 1

Call:
lm(formula = FEV ~ Smoker, data = FEVdata)

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.56614	0.03466	74.037	< 2e-16
Smokeryes	0.71072	0.10994	6.464	1.99e-10

Model 2

Call:
lm(formula = FEV ~ Smoker + Age, data = FEVdata)

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.367373	0.081436	4.511	7.65e-06
Smokeryes	-0.208995	0.080745	-2.588	0.00986
Age	0.230605	0.008184	28.176	< 2e-16

.367 = predicted FEV
non smoker age 0
-.209 = pred. 20%
lower for smoker
than non smoker of
the same age

Model 3

lm(formula = FEV ~ Smoker + Age + Smoker:Age, data = FEVdata)

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.253396	0.082651	3.066	0.00226
Smokeryes	1.943571	0.414285	4.691	3.31e-06
Age	0.242558	0.008332	29.113	< 2e-16
Smokeryes:Age	-0.162703	0.030738	-5.293	1.65e-07

at age = 0

Slope of age
for non smokers

$H_0: \beta_{\text{age} \times \text{age}} = 0$
 $H_a: \beta_{\text{age} \times \text{age}} \neq 0$

Model 4

centered

Call:
lm(formula = FEV ~ Smoker * scale(Age, center = TRUE), data = FEVdata)

$(x - \bar{x})$

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.66229	0.02305	115.490	< 2e-16
Smokeryes	0.32774	0.12861	2.548	0.0111
scale(Age, center = TRUE)	0.71650	0.02461	29.113	< 2e-16
Smokeryes:Age.centered	-0.48061	0.09080	-5.293	1.65e-07

predicted FEV for non smoker
at avg age

effect of smoking
vs. non smoking
at avg age

$$\text{cov}(u_{0j}, e_{ij}) = 0$$

$$\text{cov}(u_{0j}, u_{ij}) \neq 0$$

$$y_{ij} = \beta_{0j} + \beta_{1j} \text{NAP}_{ij} + e_{ij} \quad e_{ij} \sim N(0, \sigma^2)$$

$$\beta_{0j} = \beta_{00} + u_{0j} \quad u_{0j} \sim N(0, \tau_0^2)$$

$$\beta_{1j} = \beta_{10} + u_{1j} \quad u_{1j} \sim N(0, \tau_1^2)$$

Example 2: Beaches

Model 1

Random effects:

Groups	Name	Variance	Std.Dev.
Beach	(Intercept)	8.668	2.944
Residual		9.362	3.060

Number of obs: 45, groups: Beach, 9

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	6.5819	1.0958	6.007
NAP	-2.5684	0.4947	-5.192

τ_0^2 = beach to beach variation in intercept
 τ_1^2 = beach to beach variation in slopes

Model 2

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
Beach	(Intercept)	10.949	3.309	
	NAP	2.502	1.582	-1.00
Residual		7.174	2.678	

Number of obs: 45, groups: Beach, 9

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	6.5818	1.1883	5.539
NAP	-2.8293	0.6849	-4.131

\$Beach

	(Intercept)	NAP
1	1.7986325	-0.8597860
2	5.6926249	-2.7212003
3	-2.7426699	1.3110567
4	-2.9682494	1.4188887
5	4.5044936	-2.1532473

95% CI for β_{10}

$$y_{ij} = (\beta_{00} + u_{0j}) + (\beta_{10} + u_{1j}) \text{NAP}_{ij}$$

$$= \beta_{00} + \beta_{10} \text{NAP}_{ij} + u_{0j} + u_{1j} \text{NAP}_{ij} + e_{ij}$$

$\hat{\tau}_{01} = \hat{\rho}_{01}$ = correlation between $\hat{u}_{0j}$ & $\hat{u}_{1j}$

$$H_0: \tau_1^2 = 0 \quad (\tau_{01} = 0)$$

$$H_a: \tau_1^2 > 0$$



Model 4

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
Beach	(Intercept)	3.832	1.957	
	NAP	1.002	1.001	-1.00
Residual		7.161	2.676	

Number of obs: 45, groups: Beach, 9

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	8.9590	1.0474	8.553
NAP	-3.8812	0.7228	-5.370
ExposureCat>10	-5.3824	1.5864	-3.393
NAP:ExposureCat>10	2.4460	1.0991	2.225