

Common Misperceptions in Probability among Students in an Elementary Statistics Class

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Abstract

Students come to the study of probability with several common misperceptions. For his senior project, the senior student worked with the faculty member to design a project studying some of these misperceptions among students in an elementary statistics class. The project involved three elements: measuring the misperceptions of the students, teaching to correct these misperceptions, and measuring the improvement. The student created a survey instrument used for the pretest and the posttest from elements the ARTIST database. The three hour teaching unit used interactive activities which addressed the specific concepts on which the students performed poorly. Concurrently, the students attended lecture classes taught by the instructor on the theories of probability. After this unit, the students were tested again and the results compared with the pretest.

Introduction

This study grew out of a previous study done by the senior student on misperceptions that students bring to a probability and statistics course. In the prior study, certain misconceptions in probability were found to be prevalent among a sample of college students. This study was initiated to specifically target and eliminate two of the common misperceptions, the equiprobability bias and the representativeness misconception.

The equiprobability bias is the tendency of students to view several outcomes of an experiment as equally likely. For instance, students who have an equiprobability bias think that when two dice are rolled, all the sums possible are equally likely. They do not realize that the sum of 6 for the two dice is more probable than the sum of 2.

The representativeness misconception refers to the tendency of students to erroneously think that samples which resemble the population distribution are more probable than samples which do not. Students with this misperception will think that a sequence of coin tosses that has approximately equal numbers of heads and tails is more probable than a sequence with many more heads than tails. However, the probability of each sequence is the same. People with the representative misconception expect that a small sample

will have the at least approximately the same distribution of outcomes as the population distribution.

The group of students used in this study was a class of Math 130, Elementary Statistics, at the University of Wisconsin-Superior. The class is a requirement for students majoring in social work and legal studies, and is recommended for biology majors who do not take calculus. It also meets the liberal education mathematics requirement, and so usually has some students with other majors also. In general, these students are not strong in mathematics, but see the course as a requirement for success in their fields of study, and so they take the course seriously and work hard at it. There were 27 students who took both the pretest and the posttest. The study was designed to identify the misperceptions in this class then teach to correct these. The class consists of three 50-minute class sessions and a laboratory of 75 minutes per week. The class sessions meet in a traditional classroom, and consist primarily of lectures, question and answer sessions, and activities sessions. The laboratories meet in a computer classroom and primarily focus on learning Minitab software and reinforcing the lecture concepts. During this experiment, group activities were added to the usual computer sessions. These activities were designed to break down the misconceptions the students held at the beginning of the unit.

Methodology

This study involved three parts, a pretest, a teaching unit, and a posttest for students in a three week unit on probability in an elementary statistics class. The pretest and posttest were identical tests for ease in measuring changes. The senior student created the Probability Assessment Test (PAT) for use in this project from items in the Web ARTIST Assessment Builder (http://www.gen.umn.edu/artist/assessment_builder_intro.html) database, focusing on the specific areas of interest in the study. Eighteen multiple choice questions were chosen for the PAT from the ARTIST database. These questions allowed the senior student to assess the misperceptions of the class students in the areas of equiprobability and representativeness. Six of the questions were follow-ups to other questions, and so these questions were grouped together as 4-A and 4-B, for instance. The followup questions, when used, probed at the thinking behind the choice made for the main question. A copy of the PAT is included in the appendix.

Students in the Math 130 class took the pretest at about week 4 of the semester, before any probability had been introduced in either the class or the lab. The senior student analyzed the results of the pretests. He found that the class students did better than expected on most of the pretest questions. The percentages of students who showed correct perceptions on each of the questions are shown in Figure 1, below. As is shown in the figure, more than 50% of the students showed correct reasoning skills on all the questions except questions 5, 6, 10-A and 10-B. This shows that these students came into the class with some previous knowledge of probability, especially of the definitions and simple concepts. Questions 5, 6, and 10-A and 10-B test the equiprobability bias.

Question 5 tests both equiprobability bias and representativeness, though it is not a clear question for representativeness. This question was

- Five faces of a fair die are painted black, and one face is painted white.
 The die is rolled six times. Which of the following results is more likely?
- Black side up on five of the rolls; white side up on the other roll.
 - Black side up on all six rolls.
 - a and b are equally likely.

Only 10 of the 27 students gave the correct answer of (a). However, 9 students answered (c), showing again the equiprobability bias. The results from this question are less clear, however, as students with the representative bias would also have answered (a) also.

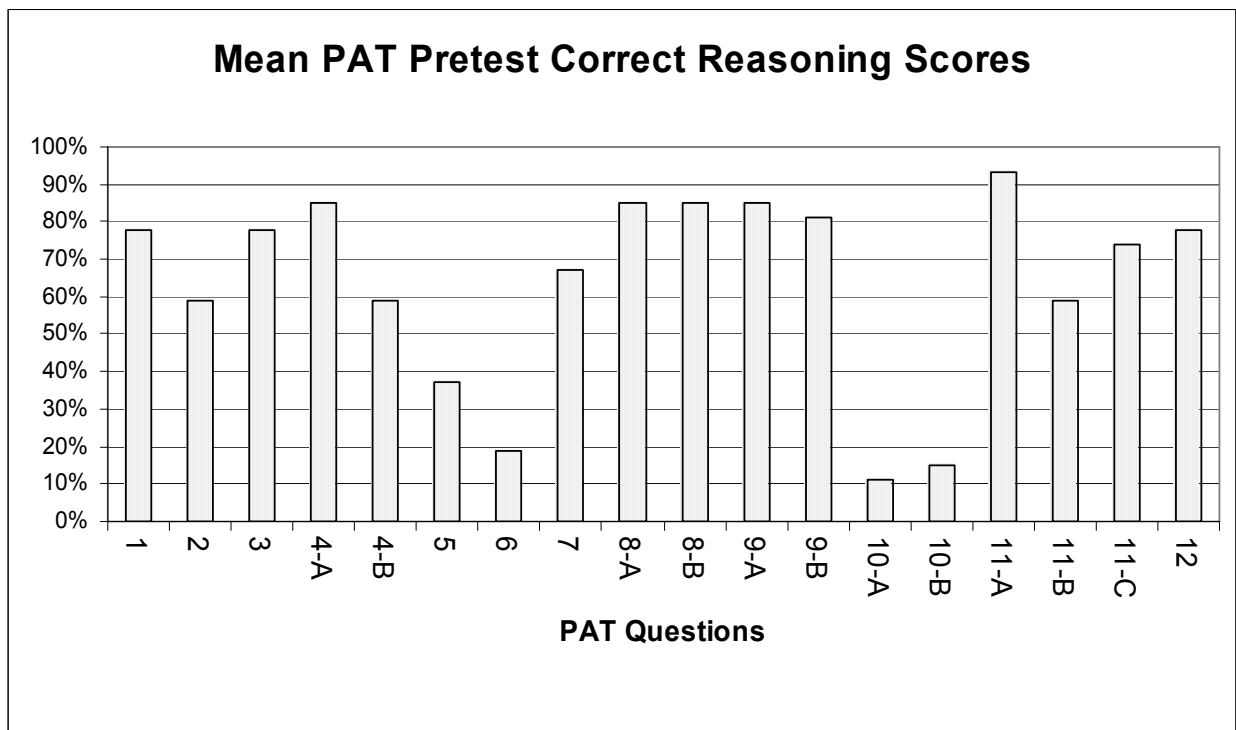


Figure 1: Percentages of students who answered each question of the PAT showing correct perceptions

Because the students knew more of representativeness than expected, the teaching lessons were planned by the senior and the faculty member to address the equiprobability concept more than the representativeness misperception, though both were covered in the lessons. Math 130 meets four times a week, three times for traditional class sessions of lecture, question and answer sessions, or activities taught by the faculty member. The fourth class meeting is a laboratory period which meets in a computer lab, and focuses on learning MiniTab and reinforcing the lecture concepts. The lab was conducted by the senior student. This class is less traditional, and the researchers decided to use two of the lab classes to do special activities to address the misconceptions of the students.

The senior student planned and taught three activities in these two lab classes. The first one involved generating sequences of coin flips by tossing a coin and then by computer simulation. The students were divided into pairs, and instructed to flip a coin 5 times each, recording the sequence of heads and tails. They then wrote their sequences on the board. When all the class sequences were on the board, the class looked for duplicate sequences and “unusual” sequences of all heads, all tails, etc. They did find both duplicate sequences and unusual sequences, thus working to dispel the representativeness misperception. They also connected their results to the law of large numbers, which had been covered in the lecture period. The students then used Minitab to model the same experiment using increasingly large samples and then compared this to the law of large numbers. The results the students found in both cases did support the theory, and also that all samples will not be representative of the population.

The second activity involved calculating the probability of an event when the probabilities of the outcomes are not equal, and when they are not known. This activity addressed the equiprobability bias. Students first predicted the probability of a Dixie cup landing on its side, top, or bottom after being tossed into the air. All the students agreed that the cup could land on its side, on the top, or on the bottom. However, some thought this made the probabilities of each landing equal to $1/3$, showing the equiprobability bias. Others thought the cup would always land on its side, so that probability would be 100%. The other estimates ranged between these. The students were then asked to do the experiment: pairs each tossed a cup into the air 50 times and recorded whether the cup did land on its side, top, or bottom. Again, the results were combined with other results from the class and the probabilities were calculated for the combined sample. The students then compared their pair’s predictions and their results with the combined results, and easily concluded that their results did not show equal probabilities. They also concluded that their samples were not always representative of the population results. (Interestingly, one group of students tossed their Dixie cups much differently than the other groups and their results were markedly different from the others. This gave us a spontaneous chance to discuss how techniques may affect results, and gave an outlier group to use for comparison.)

The third activity used modified dice, and was designed to challenge the equiprobability bias. The modified dice also would provide students with experimental results which would not agree with the theory they’d been studying in class. Students pairs were asked to roll two dice and record the sum. They did not know that one of the dice had been modified to have a 5 in place of the 2. The possible outcomes from this die were 1, 3, 4, 5, 5, or 6. This is not easily spotted, as the 2 and 5 on a die are opposite each other, so one doesn’t see them both at once. The students discussed the theory of tossing two dice and the possible sums, then tossed their dice at least 50 times, recorded the sum of the faces, and combined their data with the data from the other groups. This time, the students noticed that their results were markedly different from the theory, and that the law of large numbers was not working to make these results more like the theory. When asked what the reason might be for this to be happening, only one person in one lab looked at the dice and discovered the alteration! In the other class, the students needed to be prompted to find the answer. When the corrected probabilities were calculated, the

experimental results did approximate the theory. Students also discussed how this experiment also addresses the equiprobability bias.

After these lab classes and the standard probability unit in the lecture classes, the students were tested again using the PAT.

Results

The pretest and posttest PAT correct reasoning scores for each question are shown in the graph below, Figure 2.

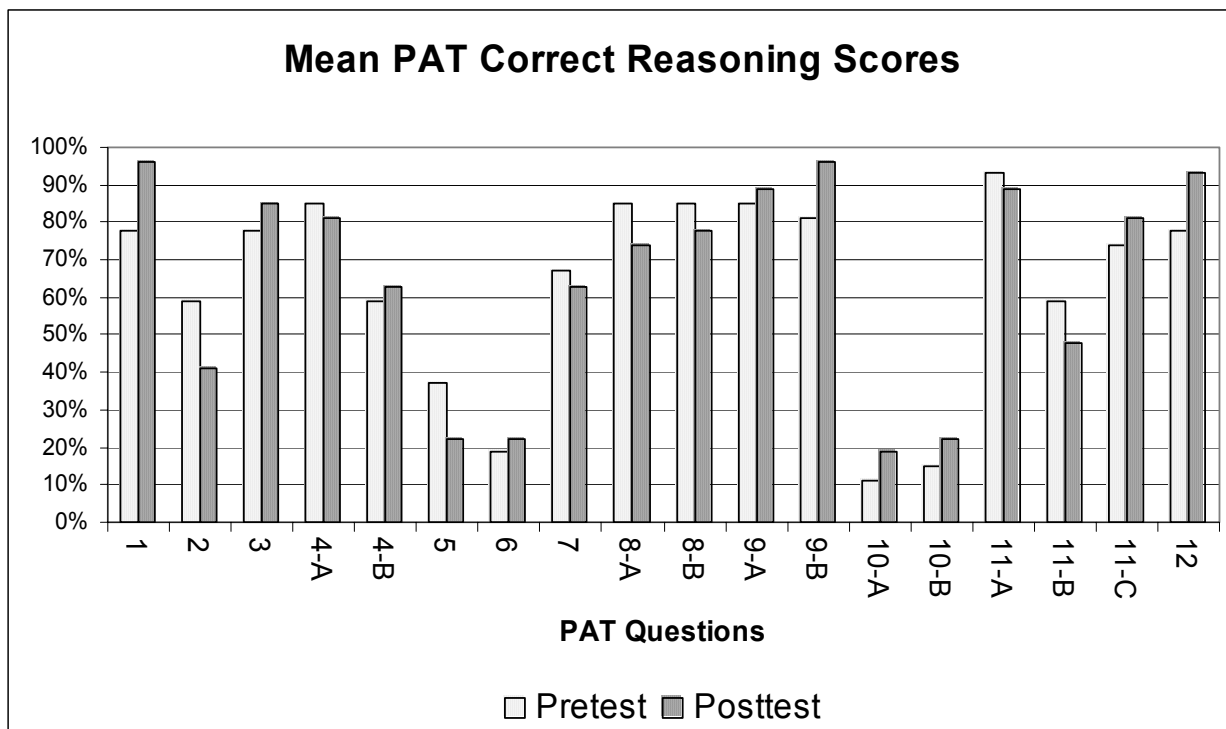


Figure 2: Percents of Correct Reasoning Answers for PAT Pretest and Posttest.

The graph shows that there are some small gains and some losses in the performance of the students on the posttest. When tested for significance, however, none of the changes were found to be significant. The group size was 27 students who took both the pretest and posttest. There are some answers to questions on the PAT which might be considered partially correct, but they were not counted here.

If one looks more closely at the test, it would appear that the students were able to respond correctly to questions which look like the book definitions or relied on the basic definitions to get the correct answer. The questions with the least success on the pretest, questions 5, 6, and 10-A and 10-B, were questions which required the students to apply the definitions and principles to slightly more complex situations. One conclusion is that students know the principles for probability and can apply these to some problems, but

have trouble when it comes to more involved problems. For example, these students correctly answered questions 4-A and 4-B. Question 4-A is

If a fair die is rolled five times, which of the following ordered sequence of results, if any, is MOST LIKELY to occur?

- a. 3 5 1 6 2
- b. 4 2 6 1 5
- c. 5 2 2 2 2
- d. Sequences (a) and (b) are equally likely.
- e. All of the above sequences are equally likely.

Twenty three of the 27 students answered correctly on the pretest, and 22 on the posttest. In the followup question, 16 or 17 answered correctly that every sequence of five rolls has exactly the same probability of occurring. However most students missed the more difficult application problem, numbers 10-A and 10-B,

When three fair dice are simultaneously thrown, which of the following results is MOST LIKELY to be obtained?

- a. Result 3: Two 5's and a 3
- b. All three results are equally likely.
- c. Result 1: a 5, a 3 and a 6 in any order.
- d. Result 2: Three 5's.

The results for problem 10-A were that 24 of the 27 on the pretest and 22 on the posttest thought that the results are equally likely. On 10-B, they thought all the results were equally unlikely.

Conclusions

The students taught probability in Math 130 during the Spring semester of 2004 came into the class with more correct basic ideas of probability than expected, as shown by their results on the pretest. This may reflect earlier coursework or basic perceptions or both. These students did have the equiprobability bias, however, and some of the representativeness misconception.

The students were given the PAT as a pretest, then three probability activities designed to address these misperceptions. After this and a three week classroom unit on probability, the students took the PAT again as a posttest.

The results of the experiment showed no significant differences between the pretest and the posttest. Partly, this is due to the high scores the students earned on the pretest. For several of the test items, there was little room for improvement. The items on which the students scored less well, the teaching did not increase the scores significantly. This might have several causes. The students might need more intensive instruction to correct this misperception. This study only used three activities and the lectures which

concurrently presented this material. Also, the activities might not have been aimed as directly at the equiprobability bias as intended. The researchers will look at these ideas in further attempts to improve the course and to correct these misperceptions. Another consideration is that this unit was only three weeks long, and might not have included enough experience for the students to absorb the new knowledge and change their perceptions of equiprobability and representativeness.

Another informal observation is that the students could answer correctly the more elementary problems presented in the PAT. They may have the beginnings of the correct concepts, yet need more involved work with the more involved problems so that they strengthen these weakly held concepts.

References

Bennett, E. (2004) *probabilistic Reasoning and Misconceptions among Elementary Statistics Students*. Unpublished Senior Project partially to fulfill the graduation requirements for the University of Wisconsin-Superior, Superior, WI.

Web ARTIST (December 18, 2003); *Web Artist Assessment Builder*. Retrieved June 27, 2004 from web site http://www.gen.umn.edu/artist/assessment_builder_intro.html

Appendix

Probability Assessment

1. Suppose a particular outcome from a random event has a probability of .02. Which of the following statements represent correct interpretations of this probability?
 - a. The outcome will never happen.
 - b. The outcome will certainly happen two times out of every 100 trials.
 - c. The outcome is expected to happen about two times out of every 100 trials.
 - d. The outcome could happen, or it couldn't, the chances of either result are the same.

2. The Springfield Meteorological Center wanted to determine the accuracy of their weather forecasts. They searched their records for those days when the forecaster had reported a 70% chance of rain. They compared these forecasts to records of whether or not it actually rained on those particular days. The forecast of 70% chance of rain can be considered very accurate if it rained on:
 - a. 95% to 100% of those days.
 - b. 85% to 94% of those days.
 - c. 75% to 84% of those days.
 - d. 65% to 74% of those days.
 - e. 55% to 64% of those days.

3. Two containers, labeled A and B, are filled with red and blue marbles in the following quantities. Each container is shaken vigorously. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win \$50. Which container gives you the best chance of drawing a blue marble?

Container	Red	Blue
A	6	4
B	60	40

- a. Container A (with 6 red and 4 blue)
- b. Container B (with 60 red and 40 blue)
- c. Equal chances from each container

- 4-A. If a fair die is rolled five times, which of the following ordered sequence of results, if any, is MOST LIKELY to occur?
- 3 5 1 6 2
 - 4 2 6 1 5
 - 5 2 2 2 2
 - Sequences (a) and (b) are equally likely.
 - All of the above sequences are equally likely.
- 4-B. Which of the following best describes the reason for your answer to the preceding question?
- Since rolling a die is random, numbers should not repeat until most of the numbers appear.
 - Every sequence of five rolls has exactly the same probability of occurring.
 - There ought to be a random mixture of numbers.
 - Any of the sequences could occur.
5. Five faces of a fair die are painted black, and one face is painted white. The die is rolled six times. Which of the following results is more likely?
- Black side up on five of the rolls; white side up on the other roll
 - Black side up on all six rolls
 - a and b are equally likely
6. When two dice are simultaneously thrown it is possible that one of the following two results occurs: Result 1: a 5 and a 6 are obtained in any order. Result 2: a 5 is obtained twice. Select the response that you agree with the most.
- The probability of obtaining each of these results is equal.
 - There is a higher probability of obtaining result 1.
 - There is a higher probability of obtaining result 2.
 - It is impossible to give an answer.

7. Students from 7 different schools were randomly sampled and asked the following: "What would you most like to do at school? A. Make good grades, B. Be good at sports, or C. Be popular?" The table presents the number of students from each school that chose each of these responses. Please answer the question by referring to the table. Students sampled from the Arles School were _____ likely to choose popularity compared to students sampled from the Brent School.

School	Grades	Sports	Popular	Total
Arles	87	33	40	160
Brent	14	11	25	50
Colst	17	6	7	30
Demot	22	16	14	52
Ernst	45	11	12	68
Fargo	30	10	20	60
Grund	28	4	17	49
Total	243	91	135	469

- a. more
- b. less
- c. equally
- d. can't tell
- 8-A. A fair coin is tossed, and it lands heads up. The coin is to be tossed a second time. What is the probability that the second toss will also be a head?
- a. $1/4$
- b. $1/2$
- c. $1/3$
- d. Slightly less than $1/2$
- e. Slightly more than $1/2$
- 8-B. Which of the following best describes the reason for your answer to the preceding question?
- a. The second toss is less likely to be heads because the first toss was heads.
- b. There are four possible outcomes when you toss a coin twice. Getting two heads is only one of them.
- c. The chance of getting heads or tails on any one toss is always $1/2$.
- d. There are three possible outcomes when you toss a coin twice. Getting two heads is only one of them.

9-A. A box contains 6 balls: 2 are red, 2 are white, and 2 are blue. Four balls are picked at random, one at a time. Each time a ball is picked, the color is recorded, and the ball is put back in the box. If the first 3 balls are red, what color is the fourth ball most likely to be?

- a. Red
- b. White
- c. Blue
- d. Blue and white are equally likely and more likely than red.
- e. Red, blue, and white are all equally likely.

9-B. Which of the following best describes the reason for your answer in the preceding question?

- a. The fourth ball should not be red because too many red ones have already been picked.
- b. The picks are independent, so every color has an equally likely chance of being picked.
- c. Red seems to be lucky.
- d. This color is just as likely as any other color.

10-A. When three fair dice are simultaneously thrown, which of the following results is MOST LIKELY to be obtained?

- a. Result 3: Two 5's and a 3
- b. All three results are equally likely.
- c. Result 1: A 5, a 3 and a 6 in any order
- d. Result 2: Three 5's

10-B. When three dice are simultaneously thrown, which of these three results is LEAST LIKELY to be obtained?

- a. Result 1: A 5, a 3 and a 6 in any order
- b. Result 2: Three 5's
- c. Result 3: Two fives and a 3 in any order
- d. All three results are equally unlikely.

11-A. Which of the following sequences is most likely to result from flipping a fair coin 5 times in a row?

- a. H H H T T
- b. T H H T H
- c. T H T T T
- d. H T H T H
- e. All four sequences are equally likely

11-B. Select one or more explanations for the answer you gave in the item above.

- a. Since the coin is fair, you ought to get roughly equal numbers of heads and tails.
- b. Since coin flipping is random, the coin ought to alternate frequently between landing heads and tails.
- c. Any of the sequences could occur.
- d. If you repeatedly flipped a coin five times, each of these sequences would occur about as often as any other sequence.
- e. If you get a couple of heads in a row, the probability of a tails on the next flip increases.
- f. Every sequence of five flips has exactly the same probability of occurring.

11-C. Which of the sequences is least likely to result from flipping a fair coin 5 times?

- a. H H H T T
- b. T H H T H
- c. T H T T T
- d. H T H T H
- e. All four sequences are equally likely

12. If you flip a fair coin and get heads 5 times in a row, what is the chance of getting tails on the next flip?

- a. Greater than 50%
- b. 50%
- c. Less than 50%